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Probabilistic representation and approximate inference of type-2 fuzzy events in Bayesian networks with interval probability parameters ☆

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ABSTRACT

It is necessary and challenging to represent the probabilities of fuzzy events and make inferences between them based on a Bayesian network. Motivated by such real applications, in this paper, we first define the interval probabilities of type-2 fuzzy events. Then, we define weak interval conditional probabilities and the corresponding probabilistic description. The expanded multiplication rule supporting interval probability reasoning. Accordingly, we propose the approach for learning the interval conditional probability parameters of a Bayesian network and the algorithm for its approximate inference. Experimental results show the feasibility of our method.

1. Introduction

As a graphical representation of probabilistic causal relationships, Bayesian networks (BNs) are effective and widely used frameworks. A Bayesian network can be constructed by means of statistical learning from sample data. Probabilistic inferences can be done by computing products of conditional probabilities from Bayesian networks (Pearl, 1988).

In real-world applications, a Bayesian network is developed to describe the causal relationships of exact sample data, while it is often desirable to determine the causality between fuzzy random events based on the Bayesian network. For example, we want to know the probability of a fuzzy event, such as "a low atmospheric pressure causes a heavy rain". For this subject, we need to discuss the following problems:

- How to represent the probability of a fuzzy event?
- How to make inferences between fuzzy events in a Bayesian network?

Actually, the traditional probability representation of fuzzy event A is

$$P(A) = \sum_{x_i \in \Omega} \mu_A(x_i) P(x_i),$$

where Ω is the sample space and μ_A is the membership function.

Note that $P(A)$ is a crisp value. It is hard to image that the uncertainty concept "heavy rain" has a crisp probability, and the reason is that we describe a linguistic variable by a certain membership function. A linguistic variable may mean different things to different people, and thus we always represent the uncertainties by a range or interval of values. Fortunately, Zadeh (1975) introduces type-2 fuzzy sets to convey the uncertainties in membership functions of ordinary fuzzy sets.

As for the probabilistic representation and inferences of random variables, in a general Bayesian network, the causal relationships among them are represented by crisp probabilities. Clearly, it is infeasible to represent and infer the probabilistic causal relationships of type-2 fuzzy events directly by the general Bayesian network. In this paper, aiming at the problems pointed out above, we focus on the representation and inference of type-2 fuzzy events in a Bayesian network.

It is known that the methods of representation and inferences of imprecise probabilities are well applied to describing the uncertainty knowledge (Walley, 1991; Cano & Moral, 2002). In recent years, credal networks, as a kind of qualitative abstraction of general Bayesian networks, have been adopted as the standard for graphical models that extend Bayesian networks to deal with imprecision with probabilities (Cozman, 2000; Cozman, 2005; Cano, Gmez, Moral, & Abelln, 2007). However, these two types of methods adopt probability intervals to represent the uncertainties of random variables. Therefore, the computation of probability intervals themselves cannot be made under a certain theoretical basis, compared to the probability computation based on the probability theory. The propagation of probability intervals during inferences cannot be guaranteed to be sound theoretically as well.

The probability theory is the foundation of Bayesian networks, and the probabilistic inferences are fulfilled based on the principles

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of the conditional probability, multiplication rule, Bayes formula, etc. To make the representation and inference of type-2 fuzzy events with a Bayesian network, it is necessary to bridge the gap between crisp probabilities and the uncertain concept. Thus we will have to extend the traditional probability theory by incorporating the characteristics of type-2 fuzzy sets. Fortunately, interval probability theory has been accepted as a formal method to representing uncertainties in an imprecise manner by typical interval values (Weichselberger, 2000; Weichselberger et al., 2003). Interval probabilities are applied to describe the semantics of imprecise probabilities and uncertain knowledge (Gilbert, de Cooman, & Kerre, 2003; Tanaka, Sugihara, & Maeda, 2004). Therefore, in this paper we will adopt the interval probability theory, instead of probability intervals, as the backbone of representation and inferences of type-2 fuzzy events with a Bayesian network.

To make the interval probability theory be suitable for the representation and inferences of type-2 fuzzy events, we first define interval probabilities, interval conditional probabilities and weak interval conditional probabilities of fuzzy events. By means of the Bayesian network with interval probabilistic parameters, the causal relationships among fuzzy events can be represented.

Further, aiming at the feasible inferences between fuzzy events in a Bayesian network, we define bound-limited weak interval conditional probabilities and expand the corresponding multiplication rule for joint probability distribution.

Based on the definitions of weak interval conditional probabilities, we mainly give a method for learning interval conditional probability tables of fuzzy random events in a Bayesian network. Then, we give a Gibbs sampling algorithm for inferences of the Bayesian network with interval probabilities. Preliminary experiments show that our method is feasible.

The remainder of this paper is organized as follows: Section 2 introduces related work. Section 3 first defines the interval probabilities, interval conditional probabilities of fuzzy events. Then, weak interval conditional probabilities, the corresponding probabilistic description, and the expanded multiplication rule are defined. Section 4 presents the method for learning interval conditional probability tables in a Bayesian network. Section 5 gives the algorithm for approximate inference in Bayesian networks with interval probability parameters. Section 6 shows the experimental results. Section 7 concludes and discusses the future work.

2. Related work

BNs have been used in many different aspects of intelligent applications (Pearl, 1988; Heckerman & Wellman, 1995). The representation and inference are universally interpreted in Pearl (1988). A lot of research has been done for constructing BNs, which is the basis of BN-related applications. Cheng proposed the method for learning the BN from data based on information theory (Cheng & Bell, 1997). Based on conditional independence, Pearl defines the Markov blanket, which describes the direct causes and direct effects given a certain node in the BN (Pearl, 1988). The discovery of Markov blanket is applied in BN's approximate reasoning, such as stochastic simulation (Pearl, 1987), variable selection for classification, causal discovery, feature selection, and so on (Margaritis & Thrun, 1999; Tsamardinos & Aliferis, 2003). However, representation and inference of fuzzy events cannot be conducted based on the general Bayesian network.

For describing high-level uncertainties, Zadeh (1975) introduces the concept of type-2 fuzzy sets, which have been successfully used in many applications, such as decision making, data pre-processing, and uncertainty knowledge modeling (Mendel, 2003). Many scientists study the set operations of membership degrees of type-2 fuzzy sets (Walker & Walker, 2005; Mizumoto & Tanaka, 1976, 1981; Karnik & Mendel, 2001). For general type-2 fuzzy sets, computational complexity is severe. Mendel (Mendel & John, 2002) derives inner and outer bounds for type-2 fuzzy sets and the computations for interval type-2 fuzzy sets are very simple, which makes this method not be suitable for the probabilistic representation of fuzzy events.

In the realm of probabilistic representation and inferences of uncertainty knowledge, many literatures discuss the inference with imprecise probabilities using graphical model (Walley, 1991), and the approximate algorithms using simulated annealing to obtain probability intervals (Cano & Moral, 2002), and so on. In recent years, credal networks, as a kind of qualitative abstraction of general Bayesian networks, have been adopted as the standard for graphical models that extend Bayesian networks to deal with imprecision with probabilities (Cozman, 2000, 2005; Cano et al., 2007). In a credal network, variables are associated with probabilistic inequities, similar to the situation that every variable in a general BN is associated with a conditional probability table (Cozman, 2000). However, the above two kinds of methods adopt probability intervals to represent the uncertainties of random variables, and thus the computations of probability intervals themselves cannot be conducted under some certain theoretical basis. Consequently, the propagation of probability intervals during inferences cannot be guaranteed to be sound by formal foundations. Therefore, in this paper we are to make representation and inferences of fuzzy event based on interval probability theory instead of probability intervals.

Many researchers are interested in representation of fuzzy measures using interval probabilities (Gilbert et al., 2003; Tanaka et al., 2004). In Weichselberger (2000), Weichselberger et al. (2003), Weichselberger gives the system of axioms describing the properties of interval probabilities and mentions two different concepts of interval conditional probability: the intuitive concept and the canonical concept. The intuitive concept of interval conditional probability is widely used as the generalization of conditional probability, but it does not satisfy the multiplication rule for joint probability distribution. The canonical conditional probability satisfies the multiplication rule, and a joint probability distribution can be reconstructed through the given conditional probabilities. Unfortunately, a canonical conditional probability cannot be interpreted as interval probability in the usual scene.

To sum up, in order to provide the effective methods for representation and inferences of causalities between fuzzy events, we extend the Bayesian network with interval probability parameters. Particularly, to make the interval probability theory be suitable for probabilistic inferences, we first give the expanded multiplication rule of bound-limited interval conditional probabilities.

3. Interval probabilities of type-2 fuzzy events

In this section, followed by the concept of type-2 fuzzy sets, we will define the interval probability, interval conditional probability, weak interval conditional probability of type-2 fuzzy events and corresponding probabilistic description, as well as the expanded multiplication rule.

3.1. The concept of type-2 fuzzy sets

A fuzzy concept is often expressed by means of a linguistic variable. A linguistic variable whose values are words or sentences in a natural language may mean different things to different people. It is difficult to determine an exact membership function for a fuzzy set that is described by a word. Zadeh (1975) introduces type-2 fuzzy sets to convey the uncertainties in membership function of ordinary fuzzy sets (henceforth called type-1 fuzzy sets).