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Classical thermodynamics and economic general equilibrium theory

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Abstract

A long history of analogy making between neoclassical economics and physical thermodynamics has unfortunately served to obscure two important relations between the two fields: their definitions of equilibria stem from essentially the same three axioms for the mathematical representations of systems, while the classes of transformation each has chosen to emphasize, and their responses to the problem of path dependence, have led them to very different interpretations of duality in those representations. Despite these conventional differences, we show that economies in which all agents have preferences quasi-linear in some good have a trading-constraint structure isomorphic to the structure of physical systems with classical thermodynamic equations of state. Exact equivalents of thermodynamic potentials, including entropy, can be constructed, and function as the economic counterparts to *free energies*. Quasi-linear economies are the most general in which the Walrasian idea of price formation as an analog of force balance can be realized. More general economic models raise the same methodological problems as more complex physical models that exhibit path-dependence. We show how the degree of aggregatability of an economic model corresponds to which properties of equilibria retain path-independence, and to the extent to which a social-welfare function exists. A new *contour money-metric utility* defines the maximal generalization of social-welfare functions in arbitrary economies, but depends on the endowments and composition of the economy in non-quasi-linear cases, and is limited to one-dimensional contours of equilibria in non-aggregatable cases. The differences between economic and thermodynamic methodology lies in the economic focus on the irreversible movement from

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initial disequilibrium endowments to equilibrium through voluntary trade, in contrast to the thermodynamic recognition that only reversible transformations lead to measurement of system structure. The consequences of respecting reversibility for economic method are sketched, and alternative interpretations of the Walrasian notion of wealth preservation are presented.

JEL classification: B0; D5; D6

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1. Introduction

The relation between economic and physical (particularly thermodynamic) concepts of equilibrium has been a topic of recurrent interest throughout the development of neoclassical economic theory. As systems for defining equilibria, proving their existence, and computing their properties, neoclassical economics (Mas-Collel et al., 1995; Varian, 1992) and classical thermodynamics (Fermi, 1956) undeniably have numerous formal and methodological similarities. Both fields seek to describe system phenomena in terms of solutions to constrained optimization problems. Both rely on *dual representations* of interacting subsystems: the state of each subsystem is represented by pairs of variables, one variable from each pair characterizing the subsystem's content, and the other characterizing the way it interacts with other subsystems. In physics the content variables are quantities like a subsystem's total energy or the volume in space it occupies; in economics they are amounts of various commodities held by agents. In physics the interaction variables are quantities like temperature and pressure that can be measured on the system boundaries; in economics they are prices that can be measured by an agent's willingness to trade one commodity for another.

The significance attached to these similarities has changed considerably, however, in the time from the first mathematical formulation of utility (Walras, 1909) to the full axiomatization of general equilibrium theory (Debreu, 1987). Léon Walras appears (Mirowski, 1989) to have conceptualized economic equilibrium as a balance of the gradients of utilities, more for the sake of similarity to the concept of force balance in mechanics, than to account for any observations about the outcomes of trade. Fisher (1892) (a student of J. Willard Gibbs) attempted to update Walrasian metaphors from mechanics to thermodynamics, but retained Walras's program of seeking an explicit parallelism between physics and economics.

As mathematical economics has become more sophisticated (Debreu, 1987) the naïve parallelism of Walras and Fisher has progressively been abandoned, and with it the sense that it matters whether neoclassical economics resembles any branch of physics. The cardinalization of utility that Walras thought of as a counterpart to energy has been discarded, apparently removing the possibility of comparing utility with any empirically measurable quantity. A long history of logically inconsistent