Finance Research Letters 7 (2010) 224–231

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Robust general equilibrium under stochastic volatility model

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A R T I C L E I N F O

Article history:
Received 3 December 2009
Accepted 14 May 2010
Available online 21 May 2010

JEL classification:
G12
G13
D58
C68

Keywords:
General equilibrium
Robust control
Stochastic volatility model
Equity premium

A B S T R A C T

This paper studies the implications of model uncertainty under stochastic volatility model for equilibrium asset pricing. We derive the equilibrium equity premium and risk-free rate in a pure-exchange economy with one representative agent who is averse not only to risk but also to model uncertainty. The results show that robustness increases the equilibrium equity premium while lowers the risk-free rate.

1. Introduction

This paper studies the implications of the presence of any uncertainty about the growth rate process for the asset pricing phenomena by adopting a general equilibrium setting with one representative agent and one perishable good. Model uncertainty, or fear of model misspecification, has been widely studied in portfolio choice and asset pricing, e.g., Epstein and Wang (1994), Anderson et al. (2003), Uppal and Wang (2003), Maenhout (2004), and Liu et al. (2005). Our approach to model uncertainty follows the line of Anderson et al. (2003) which accounts for the imprecise knowledge about the probability distribution with respect to the fundamental risks in the economy. Without considering stochastic volatility, in a pure-exchange economy with one representative agent Maenhout (2004) studied the implications of model uncertainty for equity premium in a diffusion model, while Liu et al. (2005) solved the equilibrium asset prices in a jump diffusion model by allowing model

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1544-6123/$ - see front matter
doi:10.1016/j.frl.2010.05.002

uncertainty with respect to rare events. The motivation of this study is that stochastic volatility models have been widely used in stock, bond, and currency option pricing. However, the empirical studies of Pan (2002) using time-series data from both stock and option markets indicated that the pricing kernel linking the two markets cannot be supported by such an equilibrium derived under a stochastic volatility model. Under stochastic volatility models, our study shows that, except for the market risk premium and the stochastic volatility risk premium, the equilibrium equity premium includes a third part of equity premium which is given by the uncertainty aversion of the investor.

This paper is organized as follows. A continuous time pure-exchange competitive economy with one representative agent is introduced in Section 2.1. Alternative models with respect to the reference model are introduced in Section 2.2. The equilibrium model is derived in Section 2.3. Finally, the concluding remarks are given in Section 3.

2. The model

In this section we consider a continuous time pure-exchange competitive economy with a representative agent. Our model is similar to the Lucas (1978) model, except that in this study the agent has uncertainty about the growth rate of output. Given the closed-form solutions for equilibrium consumption and portfolio decisions, the equilibrium risk-free rate and equity premium can be obtained explicitly. The treatment and notations of model uncertainty in this section will follow that of Uppal and Wang (2003) and Maenhout (2004).

2.1. The economy

We study a pure-exchange economy in which the representative agent is endowed with shares in a production technology that generates a dividend flow D_t. The economy is populated by the agent who maximizes his/her expected lifetime utility and has access to two financial assets: one riskless, paying an instantaneous rate r_t, and the other risky (equities), paying the dividend process D_t.

Consider a probability space $(\Omega, \mathscr{F}, \mathscr{P})$ endowed with a standard complete filtration $(\mathscr{F}_t | t \geqslant 0)$. We assume that D_t is a Markov process in $\mathbb{R}$ with respect to the filtration $(\mathscr{F}_t)$ in some state space $\mathfrak{D} \subset \mathbb{R}$ which follows the stochastic differential equation system

$$dD_t = \mu D_t dt + \sqrt{v_t} D_t dB_t, \tag{1}$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t}\left(\sqrt{1-\rho^2} dZ_t + \rho dB_t\right), \tag{2}$$

where B_t and Z_t are standard independent Brownian motions. This endowment flow model is the standard stochastic volatility model of Heston (1993) with constant mean growth rate $\mu \geqslant 0$ and local variance v_t.[1] v_t is a square-root mean reverting process with long-run mean θ, speed of adjustment κ, and variation coefficient σ_v of the diffusion volatility v_t. The parameters κ,θ, and σ_v are assumed to be nonnegative and satisfy $2\kappa\theta \geqslant \sigma_v^2$.

Fixing the time period at Δ, the agent is assumed to maximize the following intertemporally additive expected utility (in the absence of a preference for robustness)

$$U_t = \ln C_t \Delta + e^{-\delta t} E_t^{\mathscr{P}}[U_{t+\Delta}], \tag{3}$$

where C_t is the consumption process, and $\delta > 0$ is a subjective discount rate.

Under the measure $\mathscr{P}$, the price of the risky security follows

$$\frac{dS_t}{S_t} + \frac{D_t}{S_t} dt = \mu_S dt + \sigma_S(v_t) dB_t, \tag{4}$$

[1] Buraschi and Jiltsov (2006) and Li (2007) considered the stochastic mean growth rate, μ, by allowing investors with heterogeneous beliefs about the structure of a dividend process in a pure-exchange economy. We could also let the mean growth rate, μ, to be stochastic, and the model remains tractable and can be solved analytically. Given that our main objective is to evaluate the effect of model uncertainty on equity premium under stochastic volatility model by using robust control method, we maintain the assumption of a constant mean growth rate.