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Testable implications of general equilibrium models: An integer programming approach☆

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ABSTRACT

Focusing on the testable revealed preference restrictions on the equilibrium manifold, we show that the rationalizability problem is *NP*-complete. Subsequently, we present a mixed integer programming (MIP) approach to characterize the testable implications of general equilibrium models. Attractively, this MIP approach naturally applies to settings with any number of observations and any number of agents. This is in contrast with existing approaches in the literature. We also demonstrate the versatility of our MIP approach in terms of dealing with alternative types of assignable information. Finally, we illustrate our methodology on a data set drawn from the US economy. In this application, an important focus is on the discriminatory power of the rationalizability tests under study.

1. Motivation

We introduce a mixed integer programming (MIP) approach to verify the revealed preference characterizations of general equilibrium models. Attractively, this approach naturally deals with any number of observations and/or agents. We also present an empirical application that demonstrates the practical usefulness of our approach. To our knowledge, this is the first application that verifies the general equilibrium rationalizability conditions on a real data set. This introductory section motivates our research questions and summarizes our main contributions.

The Sonnenschein–Mantel–Debreu result can be summarized in the following way: any vector valued function of prices that satisfies Walras' law, continuity and homogeneity of degree zero is the excess demand function of some economy with at least as many agents as commodities. This celebrated result led to the rather depressing viewpoint that general equilibrium is unable to generate falsifiable predictions. From a Popperian perspective, this would label general equilibrium theory as unscientific.

However, Brown and Matzkin (1996) showed that, if we focus on the equilibrium manifold,[1] and not on the excess demand function, then the pure exchange model has strong nonparametric empirical restrictions. Toward this end, they focus on the nonparametric revealed preference implications in the tradition of Afriat (1967), Diewert (1973) and Varian (1982). Their main results characterize the finite data sets consisting of equilibrium prices, aggregate endowments and individual incomes, for which there exist continuous, concave and non-satiated utility functions such that the observed prices correspond to some equilibrium price vector for the exchange economy associated with the given endowments. More precisely, these utility functions exist if and only if there exist individual consumption bundles such that: (i) individual expenditure equals individual income, (ii) individual

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[1] The equilibrium manifold is the set of prices and individual endowments for which the excess demand function is zero.

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consumption bundles sum to aggregate endowments, and (iii) for each individual, there exists a solution for the corresponding Afriat inequalities. Moreover, they demonstrated that these restrictions are non-vacuous. If a given data set satisfies the conditions (i)–(iii), then this data set is said to be rationalizable.

Requirements (i) and (ii) are expressed as linear equalities and can therefore easily be verified by linear programming methods. Unfortunately, requirement (iii) has a quadratic form. In order to circumvent this problem, Brown and Matzkin make use of a deep result from semi-algebraic theory—the Tarski–Seidenberg theorem. This theorem states that, every first-order formula over the real field can be reduced to an equivalent quantifier-free formula. Moreover, this reduction can be established in finite time. Using this theorem, Brown and Matzkin conclude that it is possible to decide in finite time whether a solution to (i)–(iii) exists.

Subsequent research has extended the result to models including production (Carvajal, 2005), Pareto optimal provision of public goods (Snyder, 1999), financial markets (Kübler, 2003), random preferences (Carvajal, 2004), Pareto efficient and individual rational allocations (Bachmann, 2006), models with interdependent preferences (Deb, 2009) and externalities (Carvajal and Quah, 2009).[2] The usual approach adopted in these studies is as follows. First, it is demonstrated that there exist well-behaved utility functions that rationalize the data for the economy under consideration if and only if there exists some set of variables satisfying a certain collection of polynomial inequalities. Second, making use of the Tarski–Seidenberg theorem on quantifier elimination, it is inferred that the issue of rationalizability can be resolved in finite time. Third, a counterexample is provided, affirming the non-triviality of the collection of derived polynomial inequalities.

Although these results clearly demonstrate the conditions for which a given data set can belong to the equilibrium manifold, a disadvantage of this approach is that, one can only consider settings with a small number of agents and/or a limited number of observations. In their original paper, Brown and Matzkin (1996) show how to use the Tarski–Seidenberg algorithm in order to derive the testable restrictions for general equilibrium models with 2 agents and 2 observations. Unfortunately, the Tarski–Seidenberg algorithm is, for worst time complexity, at best doubly exponential in the number of quantifiers to be eliminated. Hence, the use of this approach is computationally very inefficient even for moderate sized problems (see also Brown and Matzkin, 1996 for a discussion on this issue). Most studies remain quite negligent on the issue concerning the practical verification of these conditions. A notable exception is the algorithm proposed by Brown and Kannan (2008). This algorithm enumerates every possible preference relation of all individuals over the different observations and verifies for each profile – via linear programming techniques – whether these preferences lead to a rationalization. The algorithm is exponential in both the number of observations and the number of individuals.

The computational inefficiency of the aforementioned algorithms raises the question whether there exists an algorithm that can verify the rationalizability question efficiently, i.e., in a polynomial number of steps. In Section 3, we show that, unless $P = NP$, the answer is no. In particular, the verification of restrictions of the Brown and Matzkin characterization is an *NP*-complete problem.[3] This result implies that one should not waste time trying to construct a polynomial time algorithm that verifies the rationalizability question—unless one has taken up the ambitious task of showing that $P = NP$. In turn, it gives a strong argument in favor of searching for a widely applied and 'efficient' non-polynomial time algorithm, to verify the rationalizability conditions and to open the way for introducing heuristics that can give quick (but possible inconclusive) answers.

In this paper, we suggest an easy-to-implement (non-polynomial time) procedure to check the rationalizability conditions.[4] By exploiting the equivalence between the existence of Afriat inequalities and the Generalized Axiom of Revealed Preference,[5] we show how to transform condition (iii) of Brown and Matzkin into a set of linear restrictions with mixed integer variables; i.e., we apply a mixed integer programming (MIP) procedure to characterize testable implications of general equilibrium models. Such an MIP approach has proven very useful in the literature of collective consumption models, which studies the behavior of multi-person households. See Cherchye and Vermeulen (2008) and Cherchye et al. (2011) for MIP characterizations of collective consumption models. We extend these insights to a general equilibrium setting.

From a theoretical point of view, the core motivation for adopting the MIP approach is that, it is a widely accepted and a well known approach to handle *NP*-complete problems. Besides this, we also have a number of other motivations for our MIP approach. Most notably, it allows us to avoid the use of the Tarski–Seidenberg algorithm to eliminate the quantifiers. Although in theory, this algorithm can handle data sets with any number of observations, existing applications in the context of general equilibrium models restrict their analysis to data sets with only 2 observations. Moreover, when using the Tarski–Seidenberg approach, the analysis for one general equilibrium model is not extended in a straightforward manner to another model (that accounts, for example, for different types of assignable information, public goods and/or production). Given this, an important argument pro our MIP approach is that it provides a versatile framework for analyzing testable implications of general equilibrium models. For example, in Section 3, our characterizations of general equilibrium models *à la* Brown and Matzkin naturally apply to any number of observations and agents. We further show that we can provide straightforward extensions of the basic MIP characterizations toward alternative models with different types of assignable information.[6]

We illustrate the practical usefulness of our MIP approach by means of an application to data drawn from the US economy. As indicated above, this is – as far as we know – the first application of rationalizability tests of general equilibrium models to real data. Using data on prices and aggregate consumption levels, we verify the revealed preference conditions for an economy with 8 US regions that are observed 12 times. In fact, Brown and Matzkin (1996) suggest the use of cross-sectional data for sampled regions as a typical application setting for the testable conditions under consideration. In such a setting, one can think of different agents within the same region as being of the same type, representing groups of consumers with the same tastes and incomes. We consider rationalizability in terms of alternative general equilibrium models with different degrees of assignable information. Our tests conclude that our data set is rationalizable in terms of the different models under consideration. An important concern for the type of tests that we focus on is the possible lack

[2] We refer to Carvajal et al. (2004) for an extensive overview of the literature.

[3] We refer to Garey and Johnson (1979) for an introduction into the theory of *NP*-completeness; Kalyanaraman and Umans (2008) and Talla Nobibon et al. (2011) discuss the *NP*-completeness of closely related economic models.

[4] We will only briefly touch upon the possible use of corresponding heuristics. See an earlier version of this paper (Cherchye et al., 2009) for a more detailed discussion of this issue.

[5] See Section 2 for formal definitions.

[6] In the working paper version of this paper, we also consider extensions toward models that include public goods, externalities and/or production; see Cherchye et al. (2009).