



# Risk premia in general equilibrium

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## ABSTRACT

This paper shows that non-linearities from a neoclassical production function alone can generate time-varying, asymmetric risk premia and predictability over the business cycle. These empirical key features become relevant when we allow for non-normalities in the form of rare disasters. We employ analytical solutions of dynamic stochastic general equilibrium models, including a novel solution with endogenous labor supply, to obtain closed-form expressions for the risk premium in production economies. In contrast to an endowment economy with constant investment opportunities, the curvature of the consumption function affects the risk premium in production economies through controlling the individual's effective risk aversion.

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## 1. Introduction

“... the challenge now is to understand the economic forces that determine the stochastic discount factor, or put another way, the rewards that investors demand for bearing particular risks.” (Campbell, 2000, p. 1516)

In general equilibrium models, the ‘stochastic discount factor’, i.e., the stochastic process used to discount returns of any security, is not only determined by the consumption-based first-order condition, but also linked to business cycle characteristics. In macroeconomics, dynamic stochastic general equilibrium (DSGE) models have been successful in explaining co-movements in aggregate data, but relatively less progress has been made to reconcile their asset market implications with financial data (Grinols and Turnovsky, 1993; Jermann, 1998, 2010; Tallarini, 2000; Lettau and Uhlig, 2000; Boldrin et al., 2001; Lettau, 2003; Campanale et al., 2010).<sup>1</sup> One main advantage of using general equilibrium models to explain asset market phenomena is that the asset pricing kernel is consistent with the macro dynamics.

However, surprisingly little is known about the risk premium in non-linear DSGE models, i.e., the minimum difference an individual requires to accept an uncertain rate of return, between its expected value and the certainty equivalent rate of return on saving he or she is indifferent to.<sup>2</sup> At least two primary questions present themselves. Which economic forces determine the risk premium in general equilibrium? What are the implications of using production-based models compared to the endowment economy? This paper fills the gap by studying asset pricing implications of the prototype production economy analytically. Why is this important? We argue that a clear understanding of the risk premium can

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<sup>1</sup> There is an increasing interest in macro-finance DSGE models (cf. Kaltenbrunner and Lochstoer, 2006; Gourio, 2010). A survey on the intersection of macro and finance is provided in Cochrane (2008, chapter. 7)

<sup>2</sup> Grinols and Turnovsky (1993) and Turnovsky and Bianconi (2005) study asset pricing implications of aggregate risk and/or idiosyncratic shocks in stochastic endogenous growth models with a quasi-linear production technology. Our formulation focuses on non-linear DSGE models with transitional dynamics.

best be achieved by working out analytical solutions. These solutions are shown to be important knife-edge cases which can, therefore, be used to shed light on our numerical results.

In a nutshell, this paper shows that a neoclassical production function alone generates key features of the risk premium. The economic intuition is that the individual's risk aversion, excluding singular cases, is not constant in a neoclassical production economy.

We use analytical solutions of DSGE models. For this purpose we readopt formulating models in continuous time (as in Merton, 1975; Eaton, 1981; Cox et al., 1985), which gives closed-form solutions for a broad class of models and parameter sets.<sup>3</sup> Recent research emphasizes the importance of non-normalities and non-linearities in explaining business cycle dynamics for the US economy (Fernández-Villaverde and Rubio-Ramírez, 2007; Justiniano and Primiceri, 2008; Posch, 2009). Therefore, our starting point is Lucas' fruit-tree endowment economy with non-normalities in the form of rare disasters. We obtain closed-form expressions for the risk premium from the Euler equation and relate it to the market premium over a riskless rate of return. Subsequently the framework is extended to a non-linear production economy with endogenous consumption choice and labor supply. Our approach still gives closed-form expressions under parametric restrictions.

The major findings can be summarized as follows. While the endowment economy implies a constant risk premium, non-linearities in production economies can generate time-varying, asymmetric risk premia and predictability over the business cycle.<sup>4</sup> Although these empirical key features of the risk premium are negligible in the standard real business cycle (RBC) model, we show that they become relevant when we allow for non-normalities in the form of rare disasters (Rietz, 1988; Barro, 2006, 2009). Our results are based on the finding that the 'effective risk aversion' is not constant for non-homogeneous consumption functions, as it refers to the risk aversion of the value function (cf. Carroll and Kimball, 1996).<sup>5</sup> Even for constant relative risk aversion (CRRA) of the direct utility function, the individual's effective risk aversion is not necessarily constant since it refers to gambles with respect to wealth. As we show in Section 3.2, non-homogeneous consumption functions are typically found in production economies.<sup>6</sup>

One caveat of many discrete-time models is the difficulty to obtain analytical solutions. To some extent, it is due to the difficulty of solving these models that endowment economies, in contrast to the typically non-linear production economies used in macroeconomics, are popular for asset pricing in finance. In particular by focusing on the effects of uncertainty, the traditional approach of linearization about the non-stochastic steady state does not provide an adequate framework. Alternatively, the literature suggests either risk-sensitive objectives (Hansen et al., 1999; Tallarini, 2000) or log-linearization methods (Campbell, 1994; Lettau, 2003). Similarly, numerical strategies employ perturbation and higher-order approximation (Taylor and Uhlig, 1990; Schmitt-Grohé and Uribe, 2004; Fernández-Villaverde and Rubio-Ramírez, 2006). Although these numerical methods usually are highly accurate locally, the effects of large economic shocks, such as rare disasters on approximation errors, are largely unexplored.

Our formulation of DSGE models does not suffer from such limitations. First, we use closed-form solutions for reasonable parametric restrictions to study the determinants of the risk premium analytically. Second, we make use of powerful numerical methods to examine the properties of the risk premium for a broader parameter range without relying on local approximations (cf. Posch and Trimborn, 2011). We obtain optimal consumption, optimal hours and the risk premium as functions of financial wealth in the neoclassical production economy, while our closed-form solutions can be used to gauge and ensure the accuracy of the numerical method for large economic shocks. Thus we propose this formulation as a workable paradigm in the macro-finance literature.

This paper is closely related to Lettau (2003), who derives asset pricing implications in a real business cycle model using log-linear approximations. The present paper shows that the researcher overlooks potentially important properties of the risk premium implied by the neoclassical production economy when following this approach: a log-linear approximation of the consumption function, by construction, implies a constant risk premium. As we show in Section 3.2, this property is in fact obtained for knife-edge solutions only.

Our finding about the importance of the curvature of the consumption function for the risk premium in production economies relates to Jermann (2010), who studies the properties of the risk premium as implied by producers' first-order conditions. The author identifies the curvature of adjustment costs as a key determinant of the risk premium.

There is a literature documenting that the Barro–Rietz rare disaster hypothesis generates a sizable risk premium.<sup>7</sup> The most fundamental critique, however, is on the calibration of rare disasters. Although there is empirical evidence that

<sup>3</sup> Analytical solutions to DSGE models are used in Turnovsky (1993, 2000), Corsetti (1997), Wälde (2005), Turnovsky and Smith (2006), and Posch (2009) (a detailed discussion can be found in Wälde, 2011).

<sup>4</sup> Both the time-varying feature and evidence that the risk premium increases more in 'bad times' than it decreases in 'good times' are found empirically (Lettau and Ludvigson, 2001; Mehra and Prescott, 2008).

<sup>5</sup> Any function  $f(x)$  that does not have the property that for any scalar  $b > 0$  there is a scalar  $k$  such that  $b^k f(x) = f(bx)$  is said to be non-homogeneous. A consumption function gives optimal consumption (the control variable) as a function of the state variables.

<sup>6</sup> Other contributions to Mehra and Prescott's (1985) equity premium puzzle for endowment economies, e.g., Epstein and Zin (1989); Abel (1990, 1999); Constantinides (1990); Campbell and Cochrane (1999); Veronesi (2004); Bansal and Yaron (2004), generate time-varying risk aversion through different channels.

<sup>7</sup> As a viable explanation for several macro-finance puzzles (Gabaix (2008) and Wachter (2009) suggest variable intensity versions together with recursive preferences. This not only generates a time-varying risk premium but also increases the level of the premium. A critical view is found in Julliard and Gosh (2008).

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