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# A method for solving general equilibrium models with incomplete markets and many financial assets

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ABSTRACT

This paper presents a numerical method for solving stochastic general equilibrium models with dynamic portfolio choice. The method can be applied to models with heterogeneous agents, time-varying investment opportunity sets, and incomplete asset markets. We illustrate the method using a two-country model with production. We check the accuracy of our method by comparing the numerical solution to a complete markets version of the model against its known analytic properties. We then apply the method to an incomplete markets version where no analytic solution is available. In all versions the standard accuracy tests confirm the effectiveness of our method.


## 0. Introduction

This paper presents a new numerical method for solving dynamic stochastic general equilibrium (DSGE) models with dynamic portfolio choice over many financial assets. The method can be applied to models where there are heterogeneous agents, time-varying investment opportunity sets, and incomplete asset markets. As such, our method can be used to solve models that analyze an array of important issues in international macroeconomics and finance. For example, questions concerning the role of revaluation effects in the process of external adjustment cannot be fully addressed without a model that incorporates the dynamic portfolio choices of home and foreign agents across multiple financial assets. Similarly, any theoretical assessment of the implications of greater international financial integration requires a model in which improved access to an array of financial markets has real effects; through capital deepening and/or improved risk sharing (because markets are incomplete). Indeed, there is an emerging consensus among researchers that the class of DSGE models in current use needs to be extended to include dynamic portfolio choice and incomplete markets (see, for example, Obstfeld, 2004; Gourinchas, 2006). This paper shows how an accurate approximation to the equilibrium in such models can be derived.

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We illustrate the use of our solution method by solving two versions of a canonical two-country DSGE model. The full version of the model includes production, traded and nontraded goods, and an array of equity and bond markets. Households choose between multiple assets as part of their optimal consumption and saving decisions, but only have access to a subset of the world's financial markets. As a result, there is both dynamic portfolio choice and incomplete risk-sharing in the equilibrium. We also study the equilibrium in a simplified version of the model without nontraded goods. Here households still face a dynamic portfolio choice problem but the available array of financial assets is sufficient for complete risk-sharing. We use these two versions of our model to illustrate how well our solution method works in complete and incomplete market settings; in the stationary and non-stationary environments; with log-utility and with a higher degree of risk aversion. In particular, we present several tests to show that our approximations to all sets of equilibrium dynamics are very accurate.

The presence of portfolio choice and incomplete markets in a DSGE model gives rise to a number of problems that must be addressed by any solution method. First, and foremost, the method must address the complex interactions between the real and financial sides of the economy. On the one hand, portfolio decisions affect the degree of risk-sharing which in turn affects equilibrium real allocations. On the other, real allocations affect the behavior of returns via their implications for market-clearing prices, which in turn affect portfolio choices. Second, we need to track the distribution of households' financial wealth in order to account for the wealth effects that arise when risk-sharing is incomplete. This adds to the number of state variables needed to characterize the equilibrium dynamics of the economy and hence increases the complexity of finding the equilibrium. Third, it is well-known that transitory shocks can have very persistent effects on the distribution of financial wealth when markets are incomplete, leading to non-stationary wealth dynamics in the model. Such non-stationarity is typically removed using various approaches, as discussed in details in Schmitt-Grohe and Uribe (2003). Our solution method addresses all these problems and remains accurate in both stationary and non-stationary versions of the model.

The method we propose combines a perturbation technique commonly used in solving macro-models with continuous-time approximations common in solving finance models of portfolio choice. In so doing, we contribute to the literature along several dimensions. First, relative to the finance literature, our method delivers optimal portfolios in a discrete-time general equilibrium setting in which returns are endogenously determined. It also enables us to characterize the dynamics of returns and the stochastic investment opportunity set as functions of macroeconomic state variables.[1] Second, relative to the macroeconomics literature, portfolio decisions are derived without assuming complete asset markets or constant returns to scale in production.[2]

Recent papers by Devereux and Sutherland (2010, 2011) and Tille and van Wincoop (2010) have proposed an alternative method for solving DSGE models with portfolio choice and incomplete markets.[3] Two key features differentiate their approach from the one we propose. First, their method requires at least third-order approximations to some of the model's equilibrium conditions in order to identify (first-order) variations in the portfolio holdings. By contrast, we are able to accurately characterize optimal portfolio holdings to second-order from second-order approximations of the equilibrium conditions. This difference is important when it comes to solving models with a large state space (i.e. a large number of state variables). We have applied our method to models with 8 state variables and 10 decision variables (see Evans and Hnatkovska, 2005; Hnatkovska, 2010). Second, we characterize the consumption and portfolio problem facing households using the approximations developed by Campbell et al. (2003) over the past decade. These approximations differ from those commonly used in solving DSGE models without portfolio choice, but they have proved very useful in characterizing intertemporal financial decision-making (see, for example, Campbell and Viceira, 2002). In particular, they provide simple closed-form expressions for portfolio holdings that are useful in identifying the role of different economic factors. In this sense, our approach can be viewed as an extension of the existing literature on dynamic portfolio choice to a general equilibrium setting.

The paper is structured as follows. Section 1 presents the model we use to illustrate our solution method. Section 2 describes the solution method in detail. Section 3 provides a step-by-step description of how the method is applied to our illustrative model. We present results on the accuracy of the solutions to both versions of our model in Section 3. Section 4 concludes.

## 1. The model

This section describes the discrete-time DSGE model we employ to illustrate our solution method. Our starting point is a standard international asset pricing model with production, which we extend to incorporate dynamic portfolio choice over equities and an international bond. A frictionless production world economy in this model consists of two symmetric

[1] A number of approximate solution methods have been developed in partial equilibrium frameworks. Kogan and Uppal (2002) approximate portfolio and consumption allocations around the solution for a log-investor. Barberis (2000), Brennan et al. (1997) use discrete-state approximations. Brandt et al. (2005) solve for portfolio policies by applying dynamic programming to an approximated simulated model. Brandt and Santa-Clara (2006) expand the asset space to include asset portfolios and then solve for the optimal portfolio choice in the resulting static model.

[2] Solutions to portfolio problems with complete markets are developed in Heathcote and Perri (2008), Serrat (2001), Kollmann (2006), Baxter et al. (1998), Uppal (1993), and Engel and Matsumoto (2009). Pesenti and van Wincoop (2002) analyze equilibrium portfolios in a partial equilibrium setting with incomplete markets.

[3] Ghironi et al. (2009) also develop and analyze a model with portfolio choice and incomplete asset markets. To compute the steady state asset allocations they introduce financial transaction fees. In our frictionless model portfolio holdings are derived endogenously using the conditional distributions of asset returns.