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Stochastic differential game, Esscher transform and general equilibrium under a Markovian regime-switching Lévy model

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ABSTRACT

In this paper, we discuss three different approaches to select an equivalent martingale measure for the valuation of contingent claims under a Markovian regime-switching Lévy model. These approaches are the game theoretic approach, the Esscher transformation approach and the general equilibrium approach. We employ the dynamic programming principle to derive the optimal strategies and the value functions in the stochastic differential game and the general equilibrium approaches, each of which lead to an equivalent martingale measure. We also compare equivalent martingale measures chosen by the three approaches. Under certain conditions, the equivalent martingale measures chosen by the stochastic differential game and the Esscher transformation approaches coincide. If the equity premium is in its equilibrium state, the equivalent martingale measures chosen by the Esscher transformation and the general equilibrium approaches are identical.

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1. Introduction

The valuation of contingent claims has long been an important topic in economics and finance. It plays a central role in the investment, financing and risk management activities of the financial and insurance markets around the globe. The seminal works of Black and Scholes (1973) and Merton (1973) provided a path-breaking solution to this important problem. Under the assumptions of a Geometric Brownian Motion (GBM) for the price dynamics of the underlying risky asset, a perfect market and the absence of arbitrage opportunities, they derived a preference-free, closed-form option pricing formula for a standard European call option. The pricing formula is widely adopted by market practitioners for pricing, hedging and managing risk of options. Despite its popularity, it is known that the GBM assumption for the price dynamics is not realistic and fails to incorporate many important stylized features of asset returns, such as asymmetry and heavy-tailedness of the distribution of returns, time-varying conditional volatility, sudden jumps, regime switchings and others. The Black–Scholes–Merton pricing formula also cannot explain some important empirical behaviour of option prices, namely, the implied volatility smile or smirk. Over the past few decades, various extensions to the Black–Scholes–Merton model have been introduced. These models include jump–diffusion models, GARCH models, stochastic volatility models, pure jump models, Lévy processes, regime-switching models, just to name a few.

Recently, Markovian regime-switching models have attracted a considerable interest from both researchers and practitioners in insurance and finance. The history of regime-switching models can be traced back to the early works of Quandt (1958), Goldfeld and Quandt (1973) and Tong (1978, 1983). Econometric applications of Markovian regime-switching models were later popularized by Hamilton (1989). In such models, one set of model parameters is in force at a particular time according to the state of an economy at that time. The set of model parameters will change to another set when there is a transition in the state of the underlying economy, which is usually modelled by a Markov chain. Regime-switching models provide us with a natural and convenient way to describe the effect of structural changes in different (macro)-economic conditions or different stages of business cycles. This effect is of particular importance for valuing long-dated contingent claims, such as modern insurance products with embedded options, since it is likely that economic fundamentals may change over a long period. Since the last decade or so, there have been diverse applications of regime-switching models to solving various financial problems. Some works on the use of regime-switching

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models for option pricing include Naik (1993), Guo (2001), Buffington and Elliott (2002), Boyle and Draviam (2007), Elliott et al. (2005), Siu (2005, 2008, 2011), Yuen and Yang (2010), etc.

One key economic insight behind the Black–Scholes–Merton model is the concept of the risk-neutral valuation, where the price of an option is determined as its discounted expected value under an "artificial" probability measure, namely, a risk-neutral probability measure or an equivalent martingale measure. The market considered by Black and Scholes (1973) and Merton (1973) is complete since there exists only one underlying risky asset with randomness driven by a one-dimensional Brownian motion, and thus any contingent claim can be perfectly replicated by continuously rebalancing the composition of a portfolio consisting of the risk-free asset and the underlying risky asset. However, the financial markets described by other more realistic models are mostly incomplete. As shown by Harrison and Kreps (1979) and Harrison and Pliska (1981, 1983), the market completeness is equivalent to the uniqueness of an equivalent martingale measure. So there exist infinitely many equivalent martingale measures in an incomplete financial market. A natural question is how to choose an equivalent martingale measure among infinitely many equivalent martingale measures. Different approaches have been proposed to address this problem. Recently, Øksendal and Sulem (2007a) introduced a stochastic differential game approach to choose an equivalent martingale measure for option pricing in a jump–diffusion market, where a representative agent chooses a portfolio which maximizes the expected utility of terminal wealth, while the market chooses a probability measure which minimizes this maximal expected utility. It was shown in Øksendal and Sulem (2007a) that choosing an equivalent martingale measure is an optimal strategy for the market (see also Siu, 2008 for the stochastic differential game under regime-switching models). The pioneering work by Gerber and Shiu (1994) adopted a time-honoured tool in actuarial science, namely the Esscher transform to choose an equivalent martingale measure for option valuation in an incomplete market. The use of the Esscher transform for option valuation can be justified by maximizing the expected power utility of an economic agent. Their works highlighted the interplay between the financial and actuarial pricing in incomplete markets. Applications of the Esscher transform for option pricing under regime-switching models can be found in Elliott et al. (2005), Siu and Yang (2009), Siu (2005, 2008, 2011), Elliott and Siu (2013) and others. Fu and Yang (2012) proposed a general equilibrium approach to choose an equivalent martingale measure for the price dynamics driven by a Lévy process. Many empirical features such as the negative variance risk premium, implied volatility smirk and negative skewness risk premium can be explained based on the derived equivalent martingale measure. Other works with restrictions on the distribution of the jump component on this approach include Pan (2002), Liu and Pan (2003), Liu et al. (2005) and Zhang et al. (2012).

In this paper, we investigate the game theoretic approach, the Esscher transformation approach and the general equilibrium approach to choose equivalent martingale measures for the valuation of contingent claims under a regime-switching Lévy model. A financial market consisting of a risk-free bond and a risky share is considered. The price dynamics of the risky share are governed by a Markovian regime-switching geometric Lévy process. The market interest rate, the appreciation rate, the volatility and the Lévy measure are assumed to switch over time according to a continuous-time, finite-state, observable Markov chain, whose states may represent some (macro)-economic factors (e.g. gross domestic product and purchase management index) or credit rating of a region. Firstly, we consider a two-player, zero-sum, stochastic differential game approach to choose an equivalent martingale measure for the valuation of contingent claims. Here the representative agent and the market are the two players in this game. The representative agent has a power/logarithmic utility and chooses his optimal investment–consumption strategy so as to maximize the expected, discounted utility from intertemporal consumption and terminal wealth. Whereas, the market is a fictitious player of the game and selects a real-world probability measure so as to minimize the maximal expected utility of the representative agent. We formulate this min–max problem as a stochastic differential game. We then provide a verification theorem for the Hamilton–Jacobi–Bellman–Isaac (HJBI) solution to the game and derive explicit expressions for the optimal strategies of the representative agent, the market and the value function. An equivalent martingale measure is determined by the saddle-point of the game. Secondly, we adopt a generalized version of the Esscher transform using stochastic exponentials and the Laplace cumulant process to choose an equivalent martingale measure. Thirdly, we consider a general equilibrium approach to choose an equivalent martingale measure. We formulate the general equilibrium problem of the representative agent as a stochastic optimal control problem. Then a verification theorem for the Hamilton–Jacobi–Bellman (HJB) solution to the control problem is provided. Under a market clearing condition, we derive explicit expressions for the optimal consumption rate of the representative agent, the value function and the equilibrium equity premium. Finally, we compare equivalent martingale measures chosen by the three approaches and identify the conditions under which these measures are identical.

The rest of our paper is organized as follows. In Section 2, we describe the model dynamics and formulate the optimal investment–consumption problem of the representative agent. Section 3 presents and compares three different approaches to choose equivalent martingale measures in our modelling framework. The final section concludes the paper.

2. The model dynamics

We consider a simplified, continuous-time, financial market with two primitive assets, namely, a risk-free bond and a risky share. These assets are traded continuously over time in a finite horizon $\mathcal{T} := [0, T]$, where $T < \infty$. We fix a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, which describes randomness in the market. Here $\mathcal{P}$ is a real-world probability measure or a reference probability measure from which a family of real-world probability measures is generated. We further equip $(\Omega, \mathcal{F}, \mathcal{P})$ with a right-continuous, $\mathcal{P}$-complete filtration $\mathbb{F} := \{\mathcal{F}(t)|t \in \mathcal{T}\}$, where $\mathcal{F}(t)$ is the enlarged σ-field generated by information about the values of a Brownian motion, a Poisson random measure and a Markov chain up to time t, which will be defined precisely in the later part of this section.

We model the evolution of the state of an economy over time by a continuous-time, finite-state, observable Markov chain $\mathbf{X} := \{\mathbf{X}(t)|t \in \mathcal{T}\}$ on $(\Omega, \mathcal{F}, \mathcal{P})$. As in Elliott et al. (1994), we identify the state space of the chain by a set of standard unit vectors $\mathcal{E} := \{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_N\} \subset \Re^N$, where the jth component of $\mathbf{e}_i$ is the Kronecker delta δ_{ij}, for each $i, j = 1, 2, \ldots, N$. This is usually called the canonical state space of the chain $\mathbf{X}$. To describe the statistical laws of the chain $\mathbf{X}$ under $\mathcal{P}$, we consider a constant rate matrix $\mathbf{A} := [a_{ij}]_{i,j=1,2,\ldots,N}$, where a_{ij} is the instantaneous transition rate of the chain $\mathbf{X}$ from state $\mathbf{e}_j$ to state $\mathbf{e}_i$. Note that $a_{ij} \geq 0$, for $i \neq j$ and $\sum_{i=1}^{N} a_{ij} = 0$, so $a_{ii} \leq 0$. We further assume that $a_{ij} > 0$, for each $i, j = 1, 2, \ldots, N$ with $i \neq j$, so $a_{ii} < 0$. Let $\mathbb{F}^{\mathbf{X}} := \{\mathcal{F}^{\mathbf{X}}(t)|t \in \mathcal{T}\}$ be the right-continuous, $\mathcal{P}$-complete filtration generated by the chain $\mathbf{X}$. With the canonical state space representation of $\mathbf{X}$, Elliott et al. (1994) obtained the following semimartingale dynamics for $\mathbf{X}$:

$$\mathbf{X}(t) = \mathbf{X}(0) + \int_0^t \mathbf{A}\mathbf{X}(u)du + \mathbf{M}(t), \tag{2.1}$$

where $\mathbf{M} := \{\mathbf{M}(t)|t \in \mathcal{T}\}$ is an $\Re^N$-valued, $(\mathbb{F}^{\mathbf{X}}, \mathcal{P})$-martingale.

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