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Efficient Bayesian network modeling of systems

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ABSTRACT

The Bayesian network (BN) is a convenient tool for probabilistic modeling of system performance, particularly when it is of interest to update the reliability of the system or its components in light of observed information. In this paper, BN structures for modeling the performance of systems that are defined in terms of their minimum link or cut sets are investigated. Standard BN structures that define the system node as a child of its constituent components or its minimum link/cut sets lead to converging structures, which are computationally disadvantageous and could severely hamper application of the BN to real systems. A systematic approach to defining an alternative formulation is developed that creates chain-like BN structures that are orders of magnitude more efficient, particularly in terms of computational memory demand. The formulation uses an integer optimization algorithm to identify the most efficient BN structure. Example applications demonstrate the proposed methodology and quantify the gained computational advantage.

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1. Introduction

Engineering decisions often involve probabilistic assessment of the state of a system under evolving and uncertain information. For example, in the immediate aftermath of a natural disaster, such as an earthquake affecting an urban community, decisions must be made regarding dispatch of rescue teams and inspection crews, continued operation or closure of facilities, and prioritization of repair actions and restoration of services, all of which depend on the assessment of the functioning states of various infrastructural systems. Such assessment is strongly influenced by the available information, which in the immediate aftermath of a natural disaster is highly uncertain and rapidly evolves as the states of various system components are observed or measurements of the hazard are made. Another example is the management of a deteriorating system, where decisions need to be made on the frequency and extent of inspections and on maintenance, repair and replacement actions, while future capacities and demands of the system remain uncertain. In both cases, there is need for a method to update the probabilistic assessment of the system state as information, often of uncertain type, becomes available from measurements, inspections, and other observations of the system and its components.

The Bayesian network (BN) is an ideal framework for the analysis of such systems, particularly when updating of the probabilistic model in light of evolving and uncertain information is an important objective. The BN is a graphical model consisting of nodes and directed links, which respectively represent random variables and their probabilistic dependencies [20,11]. The variables may represent the states of the components of a system, or their capacities and demands. The BN provides a convenient means for modeling dependence between the component states, which is rather difficult in most classical system reliability methods [19]. Furthermore, upon entering evidence on one or more variables, e.g., the observed states, capacities or demands of a subset of the components, the information propagates throughout the network and updates distributions of other random variables, e.g., the states of other components and the system, in accordance with the Bayes' rule. Finally, by addition of decision and utility nodes, the BN renders a decision graph that facilitates decision-making in accordance with the maximum expected utility criterion [22,11].

This paper focuses on the development of a systematic approach to using BNs for modeling the performance of systems that are defined in terms of their minimum link sets (*MLS*s) or minimum cut sets (*MCS*s). The methodology presented in this paper is motivated by our efforts in modeling the performance of spatially distributed civil infrastructure systems (e.g., a highway network or water distribution system) subjected to an earthquake hazard, with particular emphasis on post-earthquake risk assessment and decision making (see Bensi et al. [1]). For such an application, efficient computations and near real-time inference in models of large systems are essential. Furthermore, the

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considered systems are more easily characterized in terms of their *MLS*s/*MCS*s than by other means, such as fault trees, event trees or reliability block diagrams. While this paper was motivated by this specific application, the methods presented are applicable to a broader scope of problems.

The BN has been used in the past for system reliability analysis (see, e.g., [25,17,2,8,15,12,23,24]). Some of these works consider intuitive approaches to modeling systems as BNs (e.g., [8]). Others consider the fault tree (e.g., [15,2]) or the reliability block diagram (e.g., [25]) as the native source of system information. Other papers develop BNs for relatively simple systems, for which computational demands are not of particular concern. This work differs from previous efforts by defining a systematic approach to developing an efficient BN structure for modeling the reliability of complex systems when the *MLS*s or *MCS*s are the native source of system information. The approach is particularly useful when working with topologically defined systems, in which the system decomposition is commonly done through the *MLS*s and/or *MCS*s. While other authors have used BNs to model systems that are topologically defined through a reliability block diagram (e.g., [25]), no systematic attempt has been made to optimize the BN structure, particularly when working with large, multi-state systems. It turns out that conventional BN models rapidly grow in size and density with increasing size of the system, so that even for moderately sized systems the computational and memory demands make the model infeasible, especially when using exact inference algorithms with multi-state nodes. With this shortcoming in mind, in this paper we develop methods for generating efficient BN topologies for modeling systems with binary and multi-state components. A discrete optimization algorithm is developed that minimizes the density of the BN, thereby providing orders of magnitude savings in computational time and memory. This development facilitates consideration of systems, which otherwise could not be solved with conventional BN formulations.

The paper begins with a brief introduction to the BN. The introduction is limited to those aspects that are needed to motivate the remainder of the paper. Next, efficient Bayesian network formulations for modeling series and parallel systems with binary components are presented. These are then extended to general systems with binary and multi-state components. To automate construction of the efficient Bayesian network formulations, a binary integer optimization problem is formulated. Furthermore, two heuristic augmentations are presented to reduce the size of the optimization problem. Several examples demonstrate the proposed methodology and its effectiveness.

2. Brief introduction to Bayesian networks

A BN is characterized by a directed acyclic graph consisting of a set of nodes representing random variables and a set of links representing probabilistic dependencies. In this paper, we limit the treatment to BNs in which all random variables are discrete; the reader interested in BNs with continuous random variables is referred to Langseth et al. [13]. Consider the simple BN in Fig. 1. The directed links from X_1 and X_2 to X_3 indicate that the distribution of X_3 is defined conditioned on X_1 and X_2. In the BN terminology, random variable X_3 is said to be a *child* of random variables X_1 and X_2, while the latter are the *parents* of X_3. Similarly, X_4 is a child of X_1, while X_5 is a child of X_4. Each node is associated with a set of mutually exclusive and collectively exhaustive states, corresponding to the outcome space of the discrete random variable. Attached to each node is a *conditional probability table* (CPT), providing the conditional probability mass function of the random variable given each of the mutually exclusive states of its parents. For *root* nodes that have no parents, e.g., X_1 and X_2 in Fig. 1, a *marginal probability table* is assigned.

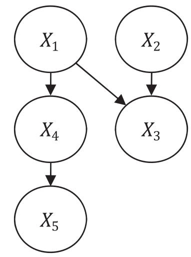

Fig. 1. A simple BN.

The joint distribution of the random variables in the BN is given as the product of the conditional distributions, i.e.,

$$p(x_1,x_2,\ldots,x_n)=\prod_{i=1}^{n} p(x_i|\mathrm{Pa}(x_i)) \tag{1}$$

where $\mathrm{Pa}(x_i)$ is the set of parents of node X_i, $p(x_i|\mathrm{Pa}(x_i))$ is the CPT of X_i and n is the number of random variables (nodes) in the BN. Thus, for the BN in Fig. 1, the joint probability mass function is

$$p(x_1,x_2,x_3,x_4,x_5)=p(x_5|x_4)p(x_4|x_1)p(x_3|x_1,x_2)p(x_1)p(x_2) \tag{2}$$

BNs are useful for answering probabilistic queries when one or more variables are observed. As an example, suppose for the BN in Fig. 1 the observations $X_3=x_3$ and $X_4=x_4$ have been made and the conditional distribution $p(x_2|x_3,x_4)$ is of interest. This posterior distribution is computed by first marginalizing the joint distribution in (2) to obtain the joint distributions of the subsets of the variables:

$$p(x_2,x_3,x_4)=\sum_{x_1,x_5} p(x_1,\ldots,x_5) \tag{3}$$

$$p(x_3,x_4)=\sum_{x_1,x_2,x_5} p(x_1,\ldots,x_5) \tag{4}$$

The desired conditional distribution then is $p(x_2|x_3,x_4)=p(x_2,x_3,x_4)/p(x_3,x_4)$. While it is possible to obtain updated distributions by this method, this is not a computationally efficient approach for non-trivial BNs. Several efficient algorithms for exact and approximate probabilistic inference in BNs have been developed (see, e.g., [3,13,14,16,26,27]). The general principles of exact inference algorithms are outlined here to highlight the requirements for efficient BN topologies.

The efficiency of the BN stems from the decomposition of the joint distribution into local conditional distributions, as exemplified in Eq. (2). When summing over the joint distribution, as in Eqs. (3) and (4), no use of the decomposition is made and computations are inefficient. However, by writing the joint distribution in the product form of Eq. (1), it is possible to rearrange the summation and product operations due to their distributive and commutative properties. As an example, Eq. (3) is written as

$$\begin{aligned} p(x_2,x_3,x_4)&=\sum_{x_1}\sum_{x_5} p(x_5|x_4)p(x_4|x_1)p(x_3|x_1,x_2)p(x_1)p(x_2) \\ &=p(x_2)\sum_{x_1} p(x_4|x_1)p(x_3|x_1,x_2)p(x_1)\sum_{x_5} p(x_5|x_4) \end{aligned} \tag{5}$$

The summation operations can be interpreted as node eliminations. Since calculations are performed from right to left, Eq. (5) corresponds to an elimination of X_5 followed by the elimination of X_1. Clearly, solving the second line of Eq. (5) is more efficient than solving the first, because the summations are performed in smaller domains. The summation over X_5 is in the domain of X_4 and X_5 only (and can actually be omitted, since it results in 1). The summation over X_1 is in the domain of X_1, X_2, X_3 and X_4, for which it is required