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Rough set theory for topological spaces

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Abstract

The topology induced by binary relations is used to generalize the basic rough set concepts. The suggested topological structure opens up the way for applying rich amount of topological facts and methods in the process of granular computing, in particular, the notion of topological membership functions is introduced that integrates the concept of rough and fuzzy sets.

Keywords: Topological space; Rough set; Fuzzy set; Membership function; Knowledge representation and processing

1. Introduction

The concept of topological structures [3] and their generalizations are one of the most powerful notions in system analysis. Many works have appeared recently for example in structural analysis [4], in chemistry [17], and physics [1]. The purpose of the present work is to put a starting point for the applications of abstract topological theory into fuzzy set theory, granular computing and rough set analysis. Fuzzy set theory appeared for the first time in 1965, in famous paper by Zadeh [19]. Since

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then a lot of fuzzy mathematics have been developed and applied to uncertainty reasoning. In this theory, concepts like fuzzy set, fuzzy subset, and fuzzy equality (between two fuzzy sets) are usually depend on the concept of numerical grades of membership. On the other hand, rough set theory, introduced by Pawlak in 1982 [13], is a mathematical tool that supports also the uncertainty reasoning but qualitatively. Their relationships have been studied in [11,12,14,18]. In this paper, we will integrate these ideas in terms of concepts in topology. Topology is a branch of mathematics, whose concepts exist not only in almost all branches of mathematics, but also in many real life applications. We believe topological structure will be an important base for knowledge extraction and processing.

2. Basic concepts

Motivation for rough set theory has come from the need to represent subsets of a universe in terms of equivalence classes of a partition of that universe. The partition characterizes a topological space, called approximation space $K = (U, R)$, where U is a set called the universe and R is an equivalence relation [7,15]. The equivalence classes of R are also known as the granules, elementary sets or blocks; we will use $R_x \subseteq U$ to denote the equivalence class containing $x \in U$. In the approximation space, we consider two operators, the upper and lower approximations of subsets: Let $X \subseteq U$.

$$\overline{R}X = \{x \in U : R_x \cap X \neq \phi\},$$

$$\underline{R}X = \{x \in U : R_x \subseteq X\}.$$

Boundary, positive and negative regions are also defined:

$$BN_R(X) = \overline{R}X - \underline{R}X,$$

$$POS_R(X) = \underline{R}X,$$

$$NEG_R(X) = U - \overline{R}X.$$

These notions can be also expressed by rough membership functions [15], namely,

$$\eta_X^R(x) = \frac{|R_x \cap X|}{|R_x|}, \quad x \in U.$$

Different values defines boundary ($0 < \eta_X^R(x) < 1$), positive ($\eta_X^R(x) = 1$) and negative ($\eta_X^R(x) = 0$) regions. The membership function is a kind of conditional probability and its value can be interpreted as a degree of certainty to which x belongs to X. A quotient set version is considered in [12,10].

Fuzzy set [19] is a way to represent populations that set theory can't describe definitely, fuzzy sets use a many (usually infinite) valued membership function, unlike classical set theory which uses a two valued membership function (i.e. an element