Available online at www.sciencedirect.com
ScienceDirect
Mathematical Social Sciences 56 (2008) 127–143
mathematical social sciences
www.elsevier.com/locate/econbase

Monotonicity properties of wholesale price contracts

Vera Tilson

Simon School of Business, University of Rochester, Rochester, NY 14627, USA

Received 12 December 2006; received in revised form 13 November 2007; accepted 11 January 2008
Available online 31 January 2008

Abstract

This paper contributes to the supply chain contracts literature in economics and operations by performing qualitative sensitivity analysis of a wholesale price contract in a two-echelon supply chain setting. Order-theory tools are used to derive sufficient conditions for monotonicity of contract parameters.

The upstream supplier is modeled as a Stackelberg leader. The supplier is assumed to have complete information about the costs and revenue function of the downstream retailer. It is shown that an equilibrium wholesale price weakly increases with an increase in the supplier production cost rate, but it may increase or decrease with an increase in the retailer cost rate. As either the supplier production cost or the retailer cost increases, the supplier profit decreases weakly. Additional sensitivity analysis is performed assuming certain properties of the retailer revenue function.

Several well-known results in the supply chain contracting literature can be considered as special cases of the more general theorems developed here. In particular, this paper reexamines the analysis of a newsvendor supply chain problem by Lariviere and Porteus [Lariviere, M.A., Porteus, E.L., 2001. Selling to the newsvendor: An analysis of price-only contracts. Manufacturing & Service Operations Management 3, 293–305]. This paper generalizes and extends their work, by establishing properties of the newsvendor demand distribution that guarantee monotonicity of the contract parameters, without requiring a unique contract solution.

JEL classification: D42

Keywords: Monotone statics analysis; Non-cooperative game; Supply chain contracting

1. Introduction

This paper examines the properties of a *wholesale price contract* between two parties, a wholesale supplier and a retailer. This type of contract (Cachon, 2003), defines an interaction

E-mail address: Vera.Tilson@simon.rochester.edu.

0165-4896/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.mathsocsci.2008.01.002

where a supplier names a wholesale price, and a retailer places an order for a quantity of goods at the named price. There are no quantity discounts offered by the supplier, and the supplier does not limit the size of the retailer's order.

Wholesale price contracts are frequently used as benchmark models to quantify the benefits of the more sophisticated "coordinating" contracts. A "coordinating" contract between a wholesaler and retailer maximizes the sum of their profits, so that the total profit of the decentralized supply chain is equal to the profit of a chain with a single decision maker. Buyback, revenue sharing, quantity flexibility contracts and others have been shown to coordinate the supply chain. Typically used for products with low variable costs, *buyback contracts* induce the retailer to order larger quantities by allowing the return of unsold products. Buyback contracts require an investment on the part of the wholesaler for processing returned inventory. With a *revenue sharing* contract, the retailer makes a lower up-front payment, and once the product is sold pays the supplier an agreed-upon portion of the revenue. An example of a successful revenue sharing contract is the agreement between the video rental chain Blockbuster and Hollywood movie studios. Demand for new movies drops precipitously over time, and wholesale prices charged by studios were too high to induce Blockbuster to stock enough copies to satisfy peak demand. Revenue sharing contracts resulted in increase in rentals due to higher availability.

The failure to coordinate the supply chain is a limitation of a wholesale price contract: each party to the contract seeks to maximize its own profit, resulting in suboptimal overall supply chain profit. The ubiquitous nature of a wholesale price contract is explained by its simplicity: the supplier need not make investment in handling retailer unsold inventory, the retailer does not need to invest in IT systems to facilitate revenue sharing. Wholesale price contracts are frequently used by small retailers for products that have low demand variability and moderate to high variable costs. A wholesale price contract is interesting in its own right because it is commonly used, and as a point of reference for other supply chain contracts.

We model the wholesale price as a two-stage, two-player sequential game of perfect and complete information. The supplier, the leader in the game, states his wholesale price, to which the retailer responds with order quantity. The retailer demand is realized after the ordered goods arrive. It is assumed that all the risk (if any) is assumed by the retailer. Such formulation of the problem allows for study of both random yield and random demand in the supply chain.

We present an analysis of monotonicity, or order-preserving, properties of such one-time contracts between a supplier and a retailer. In particular we examine, how equilibrium contract parameters change with changes in exogenous parameters affecting the retailer's revenue, capacity, or the supplier costs. There are two main reasons for our interest in monotonicity. The first one deals with managerial acceptance of proposed contract models. Managers are reluctant to accept models that qualitatively do not confirm managerial intuition. The second reason is computational: a computation of an equilibrium solution is made more efficient when it is known that a control variable changes monotonically with a change in exogenous variable. Monotone statics analysis is particularly useful in gaining insight when closed-form solutions are not available.

A frequent practice in operations literature is to use calculus tools to find a unique, or most likely equilibrium, and then to perform sensitivity analysis using differentiation. One of this paper's contributions is the elimination of the requirement for continuity of the order quantities, or uniqueness of the equilibrium. It is shown that monotonicity can be guaranteed without requiring continuity or uniqueness. Using order-theory tools allows one to consider problems where control variables and/or exogenous parameters are not continuous real numbers but rather elements of partially ordered sets. Without requiring continuity or uniqueness we discuss what