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Analysis and applications of replenishment problems under stepwise transportation costs and generalized wholesale prices

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ABSTRACT

In this study, we analyze the replenishment decision of a buyer with the objective of maximizing total expected profits. The buyer faces stepwise freight costs in inbound transportation and a hybrid wholesale price schedule given by a combination of all-units discounts with economies and diseconomies of scale. This general cost structure enables the model and the proposed solution to be also used for the supplier selection of a buyer under the single sourcing assumption. We show that the buyer's replenishment problem reduces to finding and comparing the solutions of the following two subproblems: (i) a replenishment problem involving wholesale prices given by an all-units discount schedule with economies of scale and a lower bound on the replenishment quantity, and (ii) a replenishment problem involving wholesale prices given by an all-units discount schedule with diseconomies of scale and an upper bound on the replenishment quantity. We propose solution methods for these two subproblems, each of which stands alone as practical problems, and utilize these methods to optimally solve the buyer's replenishment problem.

1. Introduction and literature

Transportation costs are one of the main cost drivers observed in supply chain management. Research on integrated transportation and production/inventory decisions shows that companies may increase total profits by simultaneous planning of transportation and production/inventory decisions (see Aucamp, 1982; Hoque and Goyal, 2000; Lee, 1986; Tersine and Barman, 1994; Toptal et al., 2003). As shipment by trucks is one of the most common transportation modes, taking into account truck capacities and costs explicitly in solving replenishment problems may lead to competitive advantages for a company. In this study, we consider a buyer subject to *full truckload shipping* as a mode of inbound transportation and a *hybrid wholesale price schedule*. This price schedule involves all-units quantity discounts with diseconomies of scale up to a certain size of order quantity, followed by all-units quantity discounts with economies of scale for larger quantities.

In truckload (TL) transportation, each additional truck requires a fixed payment as opposed to less-than-truckload (LTL) transportation in which the related costs are in proportion to the shipment quantity. Aucamp (1982), Lee (1986), Toptal et al. (2003), Toptal and Çetinkaya (2006), and Toptal (2009) are some examples of papers that model truck capacities and costs explicitly within the context of integrated replenishment and transportation decisions. Aucamp (1982) studies the classical economic order quantity (EOQ) problem assuming that the replenishment quantity is shipped via trucks having identical capacities and costs. Lee (1986) extends this study by modelling the availability of discounts on each additional truck used. Lee (1989) and Toptal et al. (2003) study the dynamic lot sizing problem and the single-warehouse, single-retailer replenishment problem, respectively, under the same transportation cost structure as in Aucamp (1982).

In comparison to the studies that consider deterministic demand (i.e., Aucamp, 1982; Lee, 1986, 1989; Toptal et al., 2003), there are also papers modelling TL shipments for inventory systems with stochastic demand (e.g., Toptal and Çetinkaya, 2006; Toptal, 2009; Ülkü and Bookbinder, 2012). Toptal and Çetinkaya (2006) study the problem of coordinating the replenishment decisions between a buyer and a vendor under transportation costs and capacities. Toptal (2009) proposes a solution for finding the order quantity that maximizes the single period expected profits of a company with stepwise freight costs and procurement costs given by an all-units discount schedule. Ülkü and Bookbinder (2012) study the shipment consolidation and pricing decisions of a manufacturer with multiple buyers who are sensitive to price and delivery time.

In this study, we consider a setting where the buyer is subject to the same freight cost structure as in Aucamp (1982). Moreover, we model a wholesale price schedule which exhibits a combination of

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economies of scale and diseconomies of scale over varying quantity intervals. There are different types of wholesale price schedules applied in practice and studied in the literature (see Benton and Park, 1996; Munson and Rosenblatt, 1998). Discount schedules with economies of scale, simply referred to as quantity discounts, are the commonly prevailing ones. Typically, in these price schedules (e.g., all-units, incremental), the unit price of an item is less for larger orders. On the other hand, in a quantity discount schedule with diseconomies of scale, the unit price of an item is more for larger orders. The changes in prices are defined by breakpoints in both of these schedules. Munson and Rosenblatt (1998) report that all-units quantity discounts, in which the discount is applied to all units in an order, is the most commonly practiced price schedule in the industry.

Quantity discounts with diseconomies of scale are also referred to as quantity premiums or quantity surcharges in the literature. Quantity premiums are common for energy products such as electricity usage and water consumption. Widrick (1985) notes that a supplier may use quantity premiums as a demarketing tool to discourage excessive consumption of a scarce resource such as water and fuel. Das (1984) discusses that this form of price schedule is also justifiable in case of limited supply, specifically, in developing economies. Quantity premiums may also be an efficient instrument for supply chain coordination when a supplier observes diseconomies of scale in replenishment costs (see Tomlin, 2003; Toptal and Çetinkaya, 2006). Tomlin (2003) studies a two-party, a manufacturer and a supplier, capacity procurement game and shows that a quantity premium cost schedule may be optimal for the manufacturer. Toptal and Çetinkaya (2006) consider a buyer–vendor system and show that when the vendor has transportation costs and capacities defined by a stepwise cost structure, quantity premiums may optimally coordinate the supply chain.

Quantity discounts are studied from the perspectives of both the buyers and the suppliers. While the former body of research focuses on the replenishment decisions under a quantity discount schedule (e.g., Abad, 1988; Arcelus and Srinivasan, 1995; Das, 1984; Hwang et al., 1990; Tersine and Barman, 1994, 1995; Toptal, 2009), the latter group investigates how a supplier should construct such price schedules (e.g., Banerjee, 1986; Lal and Staelin, 1984; Lee and Rosenblatt, 1986; Li and Liu, 2006; Monahan, 1984; Rubin and Benton, 2003; Weng, 1995). Since our focus is solving a buyer's replenishment problem under a given wholesale price schedule, our study falls into the first body of research. The wholesale price schedule considered herein is significantly different from those in earlier studies in its following feature: while the unit price of an item is more for larger orders up to a certain size, subsequent unit prices decrease with increasing order size. That is, the price schedule exhibits either diseconomies of scale or economies of scale over different quantity intervals. In order to emphasize this distinctive characteristic, we use the term "hybrid" in classifying the wholesale price schedule. It is important to note that quantity discounts with economies/diseconomies of scale are special cases of this general price schedule. There may exist several practical circumstances for a buyer to have procurement costs given by a hybrid price schedule. We show that an immediate context that a hybrid wholesale price schedule prevails is in case of supplier selection. More specifically, a profit maximizing buyer's supplier selection problem under the single sourcing assumption in a multiple suppliers setting can be modeled as a replenishment problem with a hybrid price schedule, if each supplier offers either an all-units quantity discount or an all-units quantity premium.

Two main types of supplier selection problems are identified in the literature, single-sourcing and multiple-sourcing. In single-sourcing, the purchaser is restricted to replenish from a single supplier whereas multiple-sourcing allows the replenishment quantity to be fullfilled through more than one supplier. Chaudhry et al. (1993) study a supplier selection problem allowing multiple sourcing in a setting where a supplier offers either quantity discounts or quantity premiums. Xia and Wu (2007) also consider a multiple sourcing scenario, and assume that the vendors have supply limitations and they offer quantity discounts. Swift (1995) discusses reasons why single sourcing may be preferred in practice, among which, is developing long-term cooperative relations with a supplier. We cite Aissaoui et al. (2007) for a review of studies on supplier selection.

When single sourcing is assumed in a multiple supplier setting with each supplier offering either quantity discounts or quantity premiums, a buyer's replenishment problem can be solved using one of the two methods: (i) a replenishment problem can be solved for each supplier separately and the supplier leading to the maximum expected profits can be chosen; (ii) a single schedule for wholesale prices can be constructed and a replenishment problem can be solved under this new schedule. For the problem of interest in this paper, we provide a complete analysis using both methods. The first method requires solving two types of replenishment problems; one with stepwise freight costs and quantity discounts, and one with stepwise freight costs and quantity premiums. In the setting that is of concern, the replenishment problem has a nonrecurring nature and the buyer has strictly concave production/inventory related profits for a fixed purchasing price. It is important to note that, under these considerations, the replenishment problem involving stepwise freight costs and all-units quantity discounts has been solved by Toptal (2009). To the best of our knowledge, the problem with stepwise freight costs and all-units quantity premiums has not been studied. The second method, on the other hand, requires solving a replenishment problem with stepwise freight costs and a hybrid wholesale price schedule. Again, our review of the literature suggests that this problem has not been examined.

The contributions of this paper are as follows:

(C1) As part of the first method outlined above, we solve the replenishment problem with quantity premiums and stepwise freight costs.
(C2) As part of the second method, we extend the analysis in Toptal (2009) to consider a lower bound on the order quantity. Similarly, we extend our analysis in part **(C1)** to consider an upper bound on the order quantity.
(C3) We combine the above results to work out the solution to the replenishment problem with stepwise freight costs and a hybrid wholesale price schedule.
(C4) We show that the supplier selection problem for a profit maximizing purchaser under the single sourcing assumption, reduces to the problem of interest in this paper. We also describe how to construct a single hybrid price schedule out of several price menus each of which is either an all-units quantity discount or an all-units quantity premium.

It is important to note that, although our main objective is to arrive at **(C3)**, the solutions to the subproblems described in **(C1)**–**(C2)** can be used on their own for practical purposes.

The rest of the paper is organized as follows. Section 2 presents the notation used in the paper and provides a generic mathematical formulation, which captures a wider class of problems than the specific one under consideration. In Section 3, we provide our analysis within the context of **(C1)**–**(C3)**. Section 4 follows with a detailed discussion of how a supplier selection problem under the single sourcing assumption reduces to the problem studied in this paper. In Section 5, the proposed solution methodology is illustrated over an application to the Newsvendor Model setting. Section 6 concludes the paper with a discussion on possible future research directions.