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Structural model of metacognition and knowledge of geometry

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ABSTRACT

This structural equation modeling study aimed to investigate both direct and indirect relations between metacognition and geometrical knowledge. The model was tested using data from tenth grade secondary school students (N = 923). It was used to estimate and test the hypothesized effects of two metacognitive constructs (knowledge of cognition and regulation of cognition) on three knowledge constructs (declarative, conditional, and procedural knowledge) together with the interrelationships among these three knowledge constructs. Major findings from the model indicated: (a) a reciprocal relationship existed among declarative, conditional, and procedural knowledge; (b) knowledge of cognition had a positive direct effect on procedural knowledge and a significant but negative direct effect on declarative knowledge; and (c) regulation of cognition had a positive direct effect on declarative knowledge and a significant but negative direct effect on procedural knowledge.

1. Introduction

A causal relationship between metacognition and students' mathematical knowledge has long been assumed to exist. Brown (1987) defined metacognition as "one's knowledge and control of own cognitive system". A considerable body of research has been developed to explore this relationship using correlational analysis (Lucangeli, & Cornoldi, 1997; Sperling, Howard, Miller, & Murphy, 2002; Sperling, Howard, Staley, & DuBois, 2004; Swanson, 1990; Tobias, & Everson, 2002; Veenman, Wilhelm, & Beishuizen, 2004; Veenman, Kok, & Blöte, 2005), crosstab analysis (Panaoura, Philippou, & Christou, 2003), latent variable modeling analysis (Panaoura & Philippou, 2005; Panaoura, 2007) and qualitative methods particularly interviews (Artzt & Armour-Thomas, 1992; Goos & Galbraith, 1996; Maqsud, 1997; Pugalee, 2001, 2004; Stillman & Galbraith, 1998; Wilson & Clarke, 2004). The effect of metacognitive instruction on mathematical problem solving and reasoning has also been investigated in experimental settings (Schurter, 2002; Slife, Weiss, & Bell, 1985; Kramarski, Mevarech, & Lieberman, 2001; Kramarski, Mevarech, & Arami, 2002; Kramarski, 2004; Mevarech, & Kramarski, 1997; Mevarech, 1999). Many of the afore-cited studies provide substantial evidence in favor of the positive unilateral interrelation among components of metacognition and student's mathematical knowledge. This, however, cannot explain to what extent these constructs influence one another, directly or indirectly. Veenman, Van Hout-Wolters, and Afflerbach (2006) suggested the use of PCA and LISREL analyses, which yield the best estimates among latent variables and multiple indicators. Although knowledge of cognition and regulation of cognition were suggested as the two components of metacognition (Brown, 1987), previous research particularly focused on regulation of cognition. Besides that, context of the assessments mainly focused on elementary school mathematics and rarely on secondary school mathematics, particularly concerning procedural knowledge.

The relationship between students' knowledge of concepts and procedures has also long been an important issue in the mathematics education. The interrelation among different knowledge types was particularly investigated in the domains of counting (Gelman, Meck, & Merkin, 1986), single-digit addition (Baroody & Gannon, 1984), multi-digit addition (Fuson, 1990; Hiebert & Wearne, 1996), fractions (Byrnes & Wasik, 1991; Mack, 1990; Rittle-Johnson, Siegler, & Alibali, 2001), decimal fractions (Moss & Case, 1999; Resnick et al., 1989), percent (Lembke & Reys, 1994), mathematical equivalence (Knuth, Stephens, McNeil, & Alibali, 2006; Perry, 1991; Rittle-Johnson & Alibali, 1999), linear equations (Star et al., 2005), calculus (Engelbrecht, Harding, & Potgieter, 2005), and algebra–geometry–analytic geometry (Webb, 1979). In addressing the relationship, most researchers reported that types of knowledge are learned in tandem rather than independently (Rittle-Johnson & Alibali, 1999). The topics studied in this bulk of studies have been mainly limited to elementary school mathematics, particularly arithmetics.

Researchers assessed the conceptual knowledge through tasks that involve "what" and "which" type of questions in the context of primary level of relationships (declarative knowledge), and/or that involve "how" type of questions in the context of abstract level of

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relationships (conditional knowledge) since conceptual knowledge involves building relationships between existing bits of knowledge that is comprised of primary level of relationships and abstract level of relationships. These particular constructs, however, were not classified as declarative and conditional knowledge rather introduced as conceptual knowledge. Additionally, knowledge of procedures was assessed through tasks that involve the manipulation of algorithms and procedures. Whether researchers are speaking of conceptual knowledge or procedural knowledge, they hold to the same premise that any of these types of knowledge involve declarative, conditional, and procedural knowledge (Alexander, Pate, Kullikowich, Farrell, & Wright, 1989; Ryle, 1949).

Having established these facts mentioned above, the present study aimed to test the hypothesized effects of metacognitive constructs (knowledge of cognition and regulation of cognition) on geometrical knowledge constructs (declarative, conditional, and procedural knowledge) together with the interrelationships among these knowledge constructs. We estimated the model using structural equations to assess the direct and indirect effects of the selected knowledge constructs on each other, and metacognitive constructs on knowledge of geometry. The structural relationships among these constructs were interpreted as indices of effects of one construct on the other. Thus, the purpose of this study was twofold: (a) to determine the effects of metacognitive constructs on knowledge of geometry, and (b) to determine the relationships among knowledge of geometry.

From a pedagogical point of view using knowledge of cognition and regulation of cognition can effectively inform teaching and learning. A major challenge for mathematics teachers is thus to foster the quality in thinking, assess more purposefully, and to better ends have students engaged in metacognitive processes. Students being aware of what they know can portray their learning as a transition to sense-making. If so, such metacognitive processes may offer teachers much to alleviate the understanding of the reasons underpinning students' geometrical knowledge. It has been widely acknowledged that knowledge of mathematics is energized by declarative, conditional, and procedural knowledge. Students aligned with knowledge of definitions, relational rules, and procedures are more apt to adopt what they know and do not know and use it effectively in mathematics. Similar issues of concern with corresponding inferences in other subject areas can be evident when investigating students who attach an elaborate action on their knowledge of physics, chemistry, etc. In this sense, the metacognition-knowledge model in the present study offers relations specific for mathematics as well as holds parallels and provides directions that can be specified to account for other subject areas in measures of both metacognition and knowledge.

1.1. The relationship among declarative, conditional, and procedural knowledge

Declarative knowledge (DECKNOW) forms the ground on which actions depend; conditional knowledge (CONKNOW) provides an overview that supports the connection making and assists the reconstruction of actions; procedural knowledge (PROKNOW) provides actions, changing and transforming the situations (Hiebert & Wearne, 1996; Mason & Spence, 1999). Hence, declarative knowledge refers to factual information, procedural knowledge refers to the compilation of declarative knowledge into algorithms, and conditional knowledge demands the comprehension of accessing certain facts or employ particular procedures (Alexander & Judy, 1988). An example may help us to clarify the idea of these levels. When students learn about equilateral triangle, they learn a variety of knowledge about the properties of equilateral triangles. As a declarative knowledge, they learn that an equilateral triangle has equal interior angles. At this primary level, it is usually expected that students will relate this fact to recognize the definition of an equilateral triangle. At the abstract level, the student might advance this fact to the conditional knowledge of an "if-then statement" such as "If all interior angles of a triangle are equal; then all side lengths of it are equal.". This kind of connection between the fact of equal interior angles and the fact of equal side lengths requires reflecting on the bits of information. Taken together, conceptual knowledge needs to be distinguished as declarative knowledge and conditional knowledge. To find the area of an equilateral triangle, students access procedural knowledge, such as the application of the area formula algorithm.

The stream of research on the relationships between students' knowledge of concepts and procedures reported that students' gains in procedural knowledge may lead to their gains in declarative knowledge and/or conditional knowledge (Baroody & Gannon, 1984; Gelman et al., 1986; Pesek & Kirshner, 2000; Star et al., 2005) while gains in declarative and/or conditional knowledge affect their gains in procedural knowledge (Byrnes & Wasik, 1991; Engelbrecht et al., 2005; Hiebert & Wearne, 1996; Knuth et al., 2006; Mack, 1990; Moss & Case, 1999). Students' knowledge of facts and relational rules guides their attention to relevant features of the known and unknown variables in the problem context. The organization of this knowledge leads them to generate and select appropriate procedures for solving problems, choose among alternative procedures, and transform the known procedure into a new problem situation. Students' knowledge of procedures provides the extraction of key facts and principles underlying that procedure by making attentional algorithms available. Systematic presentation of these algorithms further improves their explanations on the conceptual basis of facts and relational rules that they encounter. On the other hand, students' knowledge of facts to determine the core concepts in a problem strengthen their building relations among these core concepts and facilitate their future retrieval by the adaptation of existing links and procedures to the demands of the problem. The evidence in this causal picture led us to predict that a reciprocal relationship exists among declarative, conditional and procedural knowledge (Fig. 1).

1.2. Metacognition and knowledge of mathematics

Metacognition was primarily introduced by Flavell (1971) and generally designated as 'thinking about thinking'. Broadly speaking, it is "one's knowledge and control of own cognitive system". Brown (1987, p. 66), including two main components: knowledge of cognition (KNOOFCOG) and regulation of cognition (REGOFCOG). Flavell (1979) refers to metacognitive knowledge as person, task, and strategy; while Brown (1978) classifies it into subcomponents as declarative, conditional, and procedural knowledge. While there is consistent acknowledgement of the importance of awareness of task nature and progress, researchers mark the conceptualization of the knowledge of the personal learning characteristics. Flavell (1979)

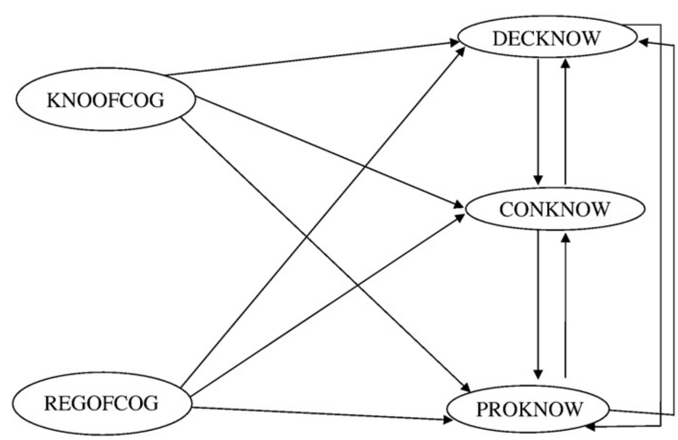

Fig. 1. The hypothesized model of metacognition and knowledge of geometry.