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Using Analogies to Overcome Student Teachers' Probability Misconceptions

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The existence of probability misconceptions at the college level and their resistance to change has been well-documented in previous research. Analogical reasoning has been successfully utilized by John Clement and associates in overcoming physics misconceptions by basing the knowledge reconstruction process on problems which draw out students' beliefs which are in agreement with accepted theory. Such problems are referred to as anchoring situations. A similar approach was attempted in this study in the area of probability.

Anchoring situations conceptually analogous to misconception-prone/target probability situations were generated. The target situations were placed in *Version A* of the *WDYTTCA* (*What Do You Think The Chances Are?*) instrument and the analogous anchoring situations were placed in *Version B*. The instrument was given to 24 secondary mathematics student teachers. *Version A* revealed that probability misconceptions were common whereas *Version B* showed that anchors for overcoming these misconceptions could be generated. Follow-up interviews with 15 volunteers indicated that the anchoring situations could be effectively utilized in overcoming probability misconceptions when the participants were engaged in a process of analogical reasoning.

In the last two decades, there have been urgent calls from various educational organizations such as the NCTM for curriculum change in mathematics. In their considered opinion, students are not being adequately prepared for the challenges of the next century. One of the recommendations which has been put forth by these educational bodies as well as by prominent individual researchers is that students need greater exposure to topics which are more relevant to the needs of everyday situations which would be encountered by a majority of students in the future (Brown, Collins & Duguid, 1989; Carl, 1989; National Research Council, 1989; Romberg, 1992). One of these topics is probability. Indeed, it is a singularly unusual day in which one does not encounter some reference to this topic such as "The probability that it will rain today is 20%" or "Joe hasn't had a hit in his last six times at bat so he's due for a hit."

Although the study of probability is highly relevant for understanding numerous everyday situations, it is also one of the topics in mathematics which is most prone to misconceptions (Shaughnessy, 1981; Hope & Kelly, 1983; Jacobsen, 1989). The latter example in the preceding paragraph illustrates one such misconception. Consequently, it becomes necessary to identify and understand these misconceptions, their sources, and how they can be overcome. This study attempted to provide further insights to these problems.

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The Purpose of the Research

The purpose of the research was to investigate secondary mathematics student teachers' conceptual understanding of situations involving probability for the ultimate purpose of overcoming the misconceptions which exist in this area. More specifically, the first objective was to generate anchoring probability situations which are conceptually isomorphic to misconception-prone/target situations. Anchoring situations are problems which are designed to draw out beliefs held by individuals which are in agreement with accepted theory and which are therefore expected to receive correct responses (Clement, 1987b). The misconception-prone/target situations used in this study were similar to those identified by numerous other researchers as frequently receiving incorrect responses, thus indicating mathematically incorrect concepts. The second objective was to utilize the anchoring situations which had been generated in attempts to overcome the probability misconceptions displayed in the target situations.

The Significance of the Study

Garfield and Ahlgren (1988) state that "... little seems to be known about how to teach probability and statistics effectively" (p. 45) and yet the study of this topic is of vital importance. A substantial portion of the difficulty in teaching the topic of probability can be attributed to the misconceptions which permeate students' thinking in this area. These have been well-documented in previous research as has their resistance to change. Attempts to overcome various mathematics misconceptions in general, and probability misconceptions in particular, have met with limited success. Clement (1987b), however, has documented remarkable results in overcoming misconceptions in physics through the use of anchors and analogical reasoning. Anchors are problems which draw out beliefs held by individuals which are in agreement with accepted theory in the particular area of study. Clement's approach thus adopted the educational tenet of beginning with what the student already knows and relating it to the misconceptions held by the student.

The use of anchors and the analogies approach to overcoming probability misconceptions had not been attempted in previous research. The only study to date which utilises some aspects of this approach is by Cox and Mouw (1992) who used four "cues added—cues removed" situations for overcoming probability misconceptions. Because of Clement's success in using analogies in physics, it seemed worthwhile to attempt his approach in the area of probability.

Overcoming probability misconceptions is an essential component of acquiring a mathematically correct understanding of probabilistic situations as they are studied in the classroom and as they occur in everyday situations. Student teachers need to overcome their own misconceptions first before they can be expected to teach effectively in the area in question. Consequently, finding a means of accomplishing this task is highly significant.

THEORETICAL BACKGROUND

Much of the original work on probability misconceptions was done by Kahneman and Tversky. They showed that these misconceptions are very common even among college students who have a statistics background. Two of the most common types of misconcep-