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Prerequisite algebra skills and associated misconceptions of middle grade students: A review

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ABSTRACT

This article provides a comprehensive literature review related to prerequisite algebra skills and associated misconceptions of middle grades students as a means to draw together ideas for research and practice. Four algebra-related content domains (Ratios and Proportional Relationships, The Number System, Expressions and Equations, and Functions) from the *Common Core State Standards for Mathematics* (CCSSM) were used as an organizing framework to examine findings that are linked to the need for greater student conceptual understanding as a priority in reaching algebra proficiency. By providing insights into prerequisite algebra skills and associated misconceptions of middle grade students, this manuscript has direct implications for classroom instruction and teacher education. This review of literature can serve as a comprehensive guide to a variety of stakeholders involved in the implementation of the middle grades algebraic content of the CCSSM (CCSSO, 2010a).

1. Overview

1.1. Introduction

The purpose of this article is to provide a review of the literature related to prerequisite algebra skills and associated misconceptions of middle grades students. We chose to focus on these skills and misconceptions because algebra often serves as a gatekeeper to success in high school, postsecondary education, and many career paths (Capraro & Joffrion, 2006; Edwards, 2000; Erbas, 2005; Stephens, 2005). The emphasis is on middle school years as that time frame is critical in preparing students for Algebra I (Capraro & Joffrion, 2006) as they make the transition from concrete to more abstract mathematics. Additionally, by linking what is known about the need for greater student conceptual understanding to the priority placed on algebra proficiency, we can begin to see how the balance between relational understanding (conceptual) and instrumental understanding (procedural) fit within the *Common Core State Standards for Mathematics* (CCSSM) (CCSSO, 2010a). Skemp (1976) brought the notion of relational understanding (conceptual) and instrumental understanding (procedural) to the forefront in his discussion of the difference between knowing how to do something and why (relational understanding) and knowing rules without reasons (instrumental understanding). He argued that relational mathematics was more advantageous because it was adaptable to new tasks, easier to remember over time, an effective goal in and of itself, and its relational schemas fostered mathematical growth.

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Table 1
Middle school CCSSM algebra-specific domains and clusters.

Level	CCSSM content domains related to algebra	Content Clusters
Sixth	Expressions and equations	Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables
Seventh	Expressions and equations	Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations
Eighth	Expressions and equations	Work with radicals and integer exponents Understand the connections between proportional relationships, lines, and linear equations Analyze and solve linear equations and pairs of simultaneous linear equations
	Functions	Define, evaluate, and compare functions Use functions to model relationships between quantities

CCSSO (2010a).

Nearly 35 years after Skemp's article on relational and instrumental understanding of mathematics, the recently released CCSSM (CCSSO, 2010a) state that a key point of the mathematics standards is to:

> ... stress not only procedural skill but also conceptual understanding, to make sure students are learning and absorbing the critical information they need to succeed at higher levels - rather than the current practices by which many students learn enough to get by on the next test, but forget it shortly thereafter, only to review again the following year. (CCSSO, 2010b, Key Points in Mathematics, para. 4)

The CCSSM (CCSSO, 2010a) and Skemp (1976) share the belief that conceptual understanding (relational understanding) of mathematics is essential to student learning. In fact, many leaders in mathematics education today support the idea that students must have a balance of both conceptual understanding and procedural fluency in all areas of mathematics, particularly algebra (Capraro & Joffrion, 2006; Fennell et al., 2007; NCTM, 2000, 2006). In this review of literature on prerequisite algebra skills and associated misconceptions, we begin with a brief overview of middle grades mathematics standards and reform, guiding frameworks, and literature search procedures. Then, using four CCSSM (CCSSO, 2010a) content domains for grades six through eight and the standards for mathematical practice as an organizing framework, we present a review of literature, followed by a discussion of implications for both research and practice.

1.2. Mathematics standards and reform

According to the National Council of Teachers of Mathematics (NCTM) Standards (2000) and the CCSSM (CCSSO, 2010a), algebraic thinking should be incorporated into all grade levels (K-12). Specifically, NCTM's *Principles and Standards for School Mathematics* (2000), the Algebra standard states:

Instructional programs from prekindergarten through grade 12 should enable all students to —

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships; and
- analyze change in various contexts. (NCTM, 2000, p. 37)

The CCSSM (CCSSO, 2010a) outlines two key points related to algebra in the middle grades: "Students who have completed 7th grade and mastered the content and skills through the 7th grade will be well-prepared for algebra in grade 8" and "The middle school standards are robust and provide a coherent and rich *preparation for high school mathematics*" (CCSSO, 2010b Key Points in Mathematics, para. 5 and para. 6). While additional prerequisite skills are needed for success in algebra (such as computational fluency with whole numbers and rational numbers), see Table 1 for the CCSSM content domains that are purely algebraic.

1.3. Guiding frameworks

In this review of prerequisite algebra skills, a variety of conceptual frameworks that align to middle school mathematics content were identified. Many of these frameworks were specific to one prerequisite skill such as Gallardo's (2002) Levels of Acceptance of Negative Numbers; Philipp's (1992a) Role of Variables; Küchemann's (1978) Variable Hierarchy; Kieran's (1992) Methods for Solving Equations; Wollman's (1983) Hierarchy of Monitoring Processes; Smith's (2008) Functional