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Grade-continuum trajectories of four known probabilistic misconceptions: What are students' perceptions of self-efficacy in completing probability tasks?

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ABSTRACT

This study introduces a new line of research that examines the cross-sectional interaction between student self-efficacy of completing probability tasks. Our study was completed in a high achieving, middle to upper middle class school district and a research university in Alabama. Through our study, we can begin to understand and organize probability misconception trajectories across grades 7, 9, 11, and 3rd year preservice mathematics teachers. In this study we examined four misconceptions: recency effects, the distinction between compound and simple events, the effect of sample size, and representativeness. Our findings indicate probability misconception trajectories slightly increase beginning in 7th grade with respect to distinguishing simple-compound events and the effect of sample size. Recency effect and representativeness misconception trajectories were found to dissipate as grade level increased. We found preservice secondary mathematics teachers to have high self-efficacy with probability misconception answers very similar to 11th graders. Recommendations include assessing for misconceptions and designing mathematics lessons and/or curricula that authentically explore these probability concepts. Preservice programs can use these findings in a manner that models authentic probability task explorations to model effective pedagogical methods. Future research on student self-efficacy with respect to mathematical misconceptions is recommended.

1. Prelude

Students do not come to the mathematics classroom with blank slates of knowledge on upcoming instruction of particular curricular topics, specifically, probability concepts. Fischbein (1987) revealed students come into classrooms with previously formed beliefs and knowledge concerning probability. In real life, many decisions are made using intuition. Students' life experiences involve basic human intuition, a necessary tool for survival (Eddy, 1982). Simmons and Nelson (2006) argue that when humans are required to make a decision, intuition is the predominant impulse that comes to mind and generally determines the response. For example, when sitting at the roulette table in a casino, a gambler is required to make a decision to place a bet. Intuition suggests that *black* is more likely to be the next outcome, given the last eight spins of the roulette wheel resulted in *red* outcomes. Thus, the gambler relying on intuition will place his bet on *black*. This real-life scenario and decision is rooted in a probability misconception. Yet misconceptions are a critical component of the learning process and knowledge construction (Smith, diSessa, & Roschelle, 1993). Chernoff (2009) indicates instead of telling a student they are

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wrong or have a misconception; educators should examine and compare a student's knowledge to the normative probability solution in the sample space. Abrahamson (2009) concludes that sample space is unexpanded by students when intuitive errors are present in probability task completion, but that students can be guided to construct correct understanding of procedures where they do not fully understand the underlying concept. If we are aware of such issues relative to probability concepts, we can improve learning trajectories that obstruct probability understanding.

Our study aimed to discover trends and interactions regarding probabilistic misconceptions among students in middle school, high school, and preservice mathematics teachers through self-reporting self-efficacy in connection with solving a probability task. To do so, we grounded our study on the teaching and learning of probability research traced back to the groundbreaking works of Daniel Kahneman and Amos Taversky in the early 1970s. Their work has shaped the last 40 years of probability education, and has been highly influential in risk-reward decision making in business, management, and economics. We build upon the work and findings of Fischbein and Schnarch (1997) among others (Borovcnik & Bentz, 1991; Fischbein, 1987, 1999; Fischbein & Gazit, 1984; Fischbein & Grossman, 1997; Fischbein, Nello, & Marino, 1991; Kahneman & Tversky, 1982a, 1982b, 1982c; Rubel, 2002). Ultimately, we extend these works by adding a new layer of understanding regarding students' perceived ability in correctly solving probability tasks to discover, at four grade levels, how strongly students' self-efficacy beliefs are potentially rooted in misconceptions and whether they change over time.

Let us assume that all probability misconceptions evolve equally in all students, and, as time progresses, students become more confident with probability tasks anchored with misconceptions as they move up the educational ladder. If this assumption were true, we would know that current curricula all result in the same problematic issue. Thus, we could easily make changes to current curricula, tailor instruction at specific times, and easily erase the origins of such issues. In the long run, probability misconceptions would cease to exist with a one-size-fits-all change. Now, let us assume this premise is false, and probability misconception trajectories are instead developmentally different among students. Further, let us suppose some probability misconceptions become more evident among students, while others dissipate and disappear. If this were true, it would imply that existing curricula and instruction are successful for some students and unsuccessful for others, even, at times, reinforcing misconceptions. What we would need to know is when failures begin to appear and if misconception trajectories are different so as to better allocate instructional time and assessment and to reshape curricula with greater emphasis at critical moments of instruction.

Our study assesses four probability misconceptions cross-sectionally, with tasks designed to detect each misconception. We sought to explore the research question: What is the relationship and interaction between 7th, 9th, and 11th grade students, and 3rd year preservice secondary mathematics teachers (grade 15), their probability misconceptions, and their perceived self-efficacy in completing probability tasks? Through student self-reporting of self-efficacy in task completion, we determine whether probability misconception trajectories change across curricular grade bands. We sought to better understand student nuance relating to probability misconceptions so as to inform educators and curriculum developers tasked with the increased expectations in the Common Core of State Standards concerning data analysis, statistics, and probability.

2. Theoretical framework

Human intuition is a double-edged sword that serves humans well in certain ecological and cultural-historical situations (Gigerenzer & Brighton, 2009; Cobb, 1989) but may hamper the development of disciplinary knowledge (Fischbein, 1975). The objective of reform-oriented mathematics education is to both elicit students' natural heuristic reasoning and help them acknowledge the ultimate limitation of this form of reasoning. While pedagogical approaches vary, some researchers have explored the prospects of helping students to both hold on to their natural heuristic reasoning and reconcile this reasoning with mathematical perspectives (Abrahamson, 2009). Heuristics reduces the complexity of decision making for humans so decisions seem more rational (Gilovich & Griffin, 2002; Kahneman & Tversky, 1972; Myers, 2007; Shaughnessy, 1977; Tversky & Kahneman, 1974). The heuristic of interest in this study was representativeness because it has long been reported in research to be commonly used in probability decision-making (Kahneman & Tversky, 1973). With representativeness, humans evaluate the probability of an event based on the degree the event is analogous to the entire possible set of outcomes *or* the degree the event is prominent in the system from which it is derived—both known to result in probabilistic misconceptions in K-12 students and the general public (Bar-Hillel, 1982; Fischbein, 1987; Gilovich & Savitsky, 2002; Kahneman & Frederick, 2002; Kahneman & Tversky, 1973; Kruglanski & Ajzen, 1983; Shaughnessy, 1977, 1981; Tversky & Kahneman, 1974). The representativeness heuristic often results in errors derived from (a) insensitivity to prior probabilities and disregard for population proportions, (b) insensitivity to the effects of sample size on predictive accuracy, (c) unwarranted confidence in a prediction based on invalid input data, (d) misconceptions of chance, (e) the illusion of validity, and (f) misconceptions of regression (Shaughnessy, 1977; Tversky & Kahneman, 1974). These errors tend to lead to the formation of Intuitive Probability Misconceptions (IPMs).

On one side of the double-edged sword, Konold (1991) indicates that students most often demonstrate knowledge of probabilistic situations through intuition. Haslanger and Saul (2006) found individuals' intuitive perceptions often are formed through naturally occurring, socially constructed experiences. As time progresses and more experiences occur, intuition can be applied to almost every daily activity in some manner. Greer (2005) argues individuals' intuitions can seriously misrepresent the actual situation. Researchers have presented the positive benefits intuition can have scientifically, such as predicting a possible experimental outcome, making comparisons between subjects or objects, generating research