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Diagnosing misconceptions: Revealing changing decimal fraction knowledge

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ABSTRACT

Conceptual change is a gradual process that occurs as students integrate new information into their existing conceptions. Throughout this process, assessing learning requires measures to diagnose misconceptions and understand how knowledge is changing. We developed three measures of misconceptions to assess students' knowledge early in instruction on decimals that measured the: 1) prevalence of misconception errors based on response patterns, 2) existence of misconceptions in a more abstract context, and 3) strength of misconceptions using confidence ratings. Students ages 9–11 ($N = 297$) completed the assessment at three time points. These measures revealed that *whole number* and *role of zero* misconceptions decreased and *fraction* misconceptions increased over time. The current measures also differentiated between weaker misconceptions that were changed after brief instruction and strongly held misconceptions. The current measures can create a more complete picture of knowledge than only measuring students' accuracy, providing a window into the conceptual change process.

1. Introduction

Researchers and teachers want to accurately measure students' knowledge, and diagnosing misconceptions is important for understanding students' changing knowledge (e.g., Resnick et al., 1989). In the current paper, "misconception" is used as a label for synthetic concepts that do not match the accepted view and that form as students attempt to integrate existing knowledge with new information, before deeper conceptual change occurs (e.g., Vamvakoussi & Vosniadou, 2010). Misconceptions can persist over a long period of time and must be overcome (Eryilmaz, 2002). For example, students have common and persistent misconceptions involving decimal magnitude, such as overgeneralizing their knowledge of whole numbers to decimal fractions (e.g., DeWolf & Vosniadou, 2014; Irwin, 2001; Resnick et al., 1989). This often occurs because students who are learning a new topic relate it to their prior knowledge. While this can be helpful for learning, it can also provide barriers for learning (e.g., Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004). Unfortunately, these misconceptions can become entrenched and adverse to change (McNeil & Alibali, 2005). To accurately assess students' learning, it is important to have knowledge measures that can diagnose misconceptions (e.g., Resnick et al., 1989). In the current study, we developed several measures for diagnosing misconceptions and assessing changing knowledge on decimal fractions. In this introduction, we outline a theory of how conceptual change occurs and how misconceptions form, the typical measures for diagnosing misconceptions, common misconceptions in the target domain of decimal fractions, and our research hypotheses.

1.1. Conceptual change and misconceptions

According to the framework theory approach to conceptual change, children first form framework theories about the world based on their everyday experiences (e.g., Vamvakoussi & Vosniadou, 2010; Vosniadou & Verschaffel, 2004). Framework theories are coherent systems of concepts that people, even young children, use to understand the world around them (e.g., Vamvakoussi & Vosniadou, 2010). These systems of concepts, including the conceptions of number, provide a principle-based system that children use to make predictions about the world and explain new information. Early on in life, children's

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conceptions of number center on natural numbers. Representational tools (e.g., finger counting) emphasize that numbers generally hold the properties of only natural numbers, such as discreteness. This conception of number as natural numbers is reinforced early in schooling as students learn addition, subtraction, and multiplication with natural numbers (Vamvakoussi & Vosniadou, 2010). Relying on this knowledge about natural numbers leads children to think that all numbers are discrete quantities and that numbers with more digits are larger. Some students also believe that adding zeros to the left-side of any number does not change its value, but adding zeros to the right-side of any number makes it larger. Through assimilation and accommodation, children add to their conceptions of number; however, when they first encounter rational numbers, the new information learned does not fit easily into their current conceptions. Consequently, the existing knowledge structure can become fragmented and synthetic concepts can form as students attempt to “assimilate … new information into their existing conceptual structures” (Stafylidou & Vosniadou, 2004, p. 505).

As students transition from their conception of number as natural numbers to a conception of number as including natural, rational, and real numbers, they generate misconceptions (i.e., synthetic concepts). Synthetic concepts “represent an intermediate state of knowledge that creates a bridge between the students' initial perspective of number and the intended scientific perspective” (Vamvakoussi & Vosniadou, 2010, p. 187). In this paper, we define “misconceptions” as “synthetic concepts”. It is important to note that misconceptions are not independent of context and that students will often hold a variety of correct concepts and misconceptions at the same time that will manifest in different ways depending on the problem or task (e.g., Vamvakoussi & Vosniadou, 2010; Vosniadou & Verschaffel, 2004). Misconceptions can become deeply entrenched, and conceptual change is a gradual, time-consuming process (e.g., McNeil & Alibali, 2005; Vamvakoussi & Vosniadou, 2010). Throughout this process of change, it is important to diagnose misconceptions and assess knowledge as a window into the conceptual change process. This will allow researchers and instructors to better understand how misconceptions form and how they change over time.

1.2. Diagnosing misconceptions

Past research has focused on diagnosing misconceptions in mathematics through individual interviews or by classifying errors on collective assessments as being associated with particular misconceptions (e.g., Resnick et al., 1989; Vamvakoussi & Vosniadou, 2010). During interviews, students are often asked to justify their answers or explain their thinking while solving problems. For example, in one study students were asked to think-aloud when answering questions such as “How many numbers are there between .005 and .006?” and to explain their answers (Vamvakoussi & Vosniadou, 2004). While interview techniques are useful for extensively investigating individuals' misconceptions, they can be time consuming and difficult to implement on a large scale or by classroom teachers. Most past research using such techniques has only included a relatively small number of participants (Irwin, 2001; Vamvakoussi & Vosniadou, 2004). An alternative method of diagnosing misconceptions involves categorizing students' errors on written assignments as misconceptions. For instance, in one study students were asked “How many numbers are there between .005 and .006?” with the multiple-choice options of there being no other number, a finite number of decimals or fractions, infinitely many decimals or fractions, or infinitely many numbers with various forms (Vamvakoussi & Vosniadou, 2010). The researchers used students' selections on these multiple-choice items to determine whether they held particular misconceptions. In another study, researchers used students' incorrect answers and calculations on word problems to categorize students' errors (Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2004). While this categorization method is easier to implement on a larger scale, it does not allow researchers or instructors to distinguish between strongly held misconceptions that are difficult to change and weakly held misconceptions that could be changed after a brief period of instruction.

In the current study, we combined three measures into an instrument that could be easily used on a larger scale and by classroom teachers to assess both the prevalence and strength of students' misconceptions. First, the *misconception error* measure categorized students' errors as particular misconceptions based on their response patterns. Second, the *confidence ratings* assessed how strongly students held these misconceptions. Third, the *general magnitude strategy* measure assessed the existence of misconceptions in the absence of other competing strategies. We wanted to develop and validate a classroom assessment using multiple methods to diagnose misconceptions. These combined measures gave a more precise and nuanced picture of students' misconceptions. We also administered the measures before and after a brief period of instruction so we could determine whether our instrument detected gradual changes in knowledge in response to new information.

1.3. Current study domain and measures

We assessed students' knowledge in the domain of decimal fractions, commonly referred to as decimals. It is important for students to master decimals to improve their learning in more advanced mathematics. For instance, mastering decimals is important for later algebra proficiency (National Mathematics Advisory Panel, 2008) and for more advanced mathematical tasks involving decimals (Hiebert & Wearne, 1985). However, children and adults often have difficulty understanding decimals (e.g., Glasgow, Ragan, Fields, Reys, & Wasman, 2000; Rittle-Johnson, Siegler, & Alibali, 2001; Stacey et al., 2001). For instance, 67% of Grade 4 students in the US could not solve a problem correctly that involved placing 1.7 on a number line from 0 to 3 (National Center for Education Statistics, 2011).

Such difficulties with decimals often stem from common and persistent difficulties transitioning from a conception of number as natural numbers to a conception of number as including natural, rational, and real numbers. During this conceptual change process, students often generate several misconceptions involving decimal magnitude (e.g., Desmet, Gregoire, & Mussolin, 2010; Glasgow et al., 2000; Irwin, 2001; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985; Stacey et al., 2001). Three common misconceptions, as defined by Resnick et al. (1989) and Irwin (2001), are: 1) the *whole number* misconception, 2) the *role of zero* misconception, and 3) the *fraction* misconception. First, the *whole number* misconception involves thinking of decimals as if they contain all the properties of whole numbers (e.g., thinking .25 is greater than .7 because 25 is greater than 7). Students incorrectly apply their generally well-developed knowledge about whole numbers to decimals, sometimes referred to as the whole number bias (Ni & Zhou, 2005). This seems to reflect attempts to assimilate decimals into a conception of numbers as whole numbers. Second, a related misconception specifically involves the *role of zero*, which is related to students' overgeneralization of a particular property of whole numbers to decimals. When a zero is in the tenths place, students often ignore it and treat the following digit as if it is in the tenths place (e.g., .07 is the same as .7). In addition, students assume that adding a zero on the end of a decimal increases its magnitude (e.g.,