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A model of partnership formation☆

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ABSTRACT

This paper presents a model of partnership formation. A number of agents want to conduct some business or other activities. Agents may act alone or seek a partner for cooperation and need in the latter case to consider with whom to cooperate and how to share the profit in a collaborative and competitive environment. We provide necessary and sufficient conditions under which an equilibrium exists. In equilibrium, the partnership formation and the payoff distribution are endogenously determined. Every agent realizes his full potential and has no incentive to deviate from either staying independent or from the endogenously determined partner and payoff. The partnership formation problem contains the widely studied assignment market problem as a special case.

1. Introduction

Partnership is one of the most common and fundamental relation patterns in society. This paper addresses a partnership formation problem. A group of agents wish to conduct business or some other activities. Each agent may act alone or seek a partner for cooperation. When an agent acts as a sole proprietor, he gains a certain payoff by himself, which could be his outside option. When an agent cooperates with another agent, they obtain a joint payoff and this value has to be divided between the two partners. Different partners generate different joint payoffs and thus may lead to a different payoff share for an agent. In this collaborative and competitive environment, each agent has to evaluate what is more profitable, acting alone or seeking a partner. In the latter case, he has to decide with whom to cooperate and how to share the joint payoff in a satisfactory way. We may imagine that after an initial period of negotiation and bargaining, a number of partners and independents will be formed. Under proper circumstances, this process will reach an equilibrium state in which every agent is satisfied in the sense that no further more favorable deals could be obtained. In equilibrium, all agents realize their full potential and have no incentive to deviate from either staying independent or cooperating with the endogenously determined partner. Many practical and well-known problems fit into this framework. To name but a few, in the professional tennis competition, there are parallel singles and double tournaments. The players can freely form pairs for the doubles and are competing for a variety of prizes; see Eriksson and Karlander (2001). Another typical instance is the roommate problem with sharing cost. The famous assignment market is also an important and interesting example.

Our analysis is closely related to the models on assignment markets studied by Koopmans and Beckmann (1957), Shapley and Shubik (1972), Shapley and Scarf (1974), Crawford and Knoer (1981), Kelso and Crawford (1982), Svensson (1983), Quinzii (1984), Demange et al. (1986), Kaneko and Yamamoto (1986), and Yamamoto (1987). In their classic papers, Koopmans and Beckmann (1957) and Shapley and Shubik (1972) investigate assignment markets from the viewpoint of equilibrium theory and cooperative game theory, respectively. In such markets, there are many buyers and sellers, and transactions are bilateral, namely, bring together a buyer and a seller of a single commodity. By means of the Birkhoff–von Neumann theorem on doubly stochastic matrices and duality theory in linear programming it is shown that the set of Walrasian equilibrium price vectors is a nonempty

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lattice and coincides with the core. Shapley and Scarf (1974) consider a swap market model without monetary transfers and show the existence of a core allocation. Svensson (1983), Quinzii (1984), Kaneko and Yamamoto (1986) and Yamamoto (1987) extend the models of Koopmans and Beckmann (1957) and Shapley and Shubik (1972) by allowing nonlinear utilities in both money and items.

Crawford and Knoer (1981) and Demange et al. (1986) propose price adjustment processes for the assignment markets studied by Koopmans and Beckmann (1957), Shapley and Shubik (1972) and prove that their processes converge to an equilibrium. Kelso and Crawford (1982) examine a job assignment model in which each firm can hire many workers but each worker is allowed to work only at one firm. They prove through a salary adjustment process that there exists an equilibrium, if every firm views all workers as substitutes. Other related models can be found in the literature on auction and matching; see for instance, Roth and Sotomayor (1990), Gul and Stacchetti (1999), Gul and Stacchetti (2000), Milgrom (2000), Ausubel (2006), Sun and Yang (2006) and Ostrovsky (2008).

This paper examines a partnership formation problem that subsumes and extends the assignment models studied by Koopmans and Beckmann (1957), Shapley and Shubik (1972), Crawford and Knoer (1981), Demange et al. (1986) among others. In these assignment models the role of an agent is exogenously given and each agent is either a buyer (firm) or a seller (worker). So, all agents are exogenously split into two disjoint groups. Agents in the same group do not have any involvement with each other and cannot work together as partners. In the current model, the role of an agent need not be exogenously given and each agent may stay alone or work together with someone else. Precisely, because it is possible for an agent to form a partnership with anyone else, this creates not only more opportunities for agents to form partners but also more obstacles to cooperate. We provide necessary and sufficient conditions for the existence of an equilibrium in the partnership formation problem. More importantly, we also give a complete characterization of the set of solutions to the problem[1] and offer a general and intuitive sufficient condition (Assumption 1). This condition is always satisfied for the assignment market models, and so it explains why the assignment models always possess an equilibrium.

In the partnership formation problem, permitted coalitions only consist of at most two individuals. Simple as they are, such coalitions are compelling, relatively easy to form, and widely observed in real life. For example, most transactions, trade, and merger occur bilaterally. Furthermore, the conditions for equilibrium existence are rather mild and natural. In addition, an analysis on the partnership formation may yield useful insights into practical situations how stable partnerships can be built and be also a necessary step to study more general coalition formation problems. From a different perspective, the following three papers deal with the latter problem. Hart and Kurz (1983) examine a general coalition formation problem. Assuming that players' prospects in various coalition structures are evaluated by a coalition structure value, they study stable coalition structures using a strategic form game. Aumann and Myerson (1988) investigate endogenous formation of cooperation structure under which players' payoffs follow the Myerson value, i.e., the Shapley value in graph games. Qin (1996) considers a cooperation-formation game in which players choose independently with whom they wish to cooperate in a given coalitional game, and players' payoffs are specified by a solution imposed on the coalitional game. He shows how cooperation evolves under best-response and fictitious-play learning processes.

The current model is most closely related to Eriksson and Karlander (2001). They examine a similar model in the context of roommate matching and introduce different sufficient conditions for equilibrium existence based on a graph theoretic approach. Klaus and Nichifor (2009) analyze properties of solutions to one-sided assignment problems, including consistency and pairwise-monotonicity. Furthermore, our model is also closely related to partitioning games (see Kaneko and Wooders, 1982; Qin, 1996) and to NTU games (see Predtetchinski and Herings, 2004). However, due to the special structure of the current model, it is possible to provide intuitive and well-behaved conditions like Assumption 1 for the existence of equilibrium.

The rest of the paper proceeds as follows. In Section 2 we present the model. In Sections 3 we establish all existence results. In Section 4 we conclude.

2. The model

Suppose there are n agents and let $N = \{1, 2, \ldots, n\}$ denote the set of agents. We assume that agents seek a partner to cooperate or act alone to obtain payoff. If two agents i and j in N, $j \neq i$, cooperate, they make a joint payoff of $v(\{i, j\})$. The value $v(\{i, j\})$ may differ for different pairs i and j. If agent $i \in N$ acts alone, he will have a payoff of $v(\{i\})$. This value could be his outside option. We call $v(\cdot)$ the *value function*. For any positive integer k, let I_k denote the set $\{1, \ldots, k\}$.

Definition 1. An *assignment* on N is a partition $P = \{U_1, \ldots, U_k\}$ of N satisfying that $|U_h| \leq 2$ for every $h \in I_k$.

When, for an assignment $P = \{U_1, \ldots, U_k\}$ on N, $|U_h| = 2$ for some $h \in I_k$, we say that the two agents in U_h are partner of each other or are being matched in P, and when $|U_h| = 1$, we say that the single agent in U_h is a sole proprietor or an independent in P. An assignment is a complete matching if every agent has a partner. A payoff vector is a vector $r \in \mathbb{R}^n$ with r_i the payoff or revenue of player $i \in N$.

In this economic environment, while bearing in mind that any other agent could be his partner or his competitor, every agent has to contemplate whether to act alone and take as payoff his value or to seek a partner and share with this partner the joint payoff. An agreement or contract between two agents, who become partners, should specify how the joint payoff will be divided. Of course, a rational agent will not rush to form a partnership with another agent merely because this agent could offer him immediately a payoff share higher than his own value. He will instead try to squeeze payoff gain as much as possible and ultimately enter a partnership if he does with someone when he has contented himself that no better contracts could be obtained with any other agent. We look for an allocation at which every agent will be satisfied with his choice to stay independent or to sign a contract for cooperation with a specific partner and obtain a payoff share and therefore has no incentive to deviate. This problem is called the *partnership formation problem* and is denoted by (N, v).

Definition 2. An *allocation* (P, r) for the partnership formation problem (N, v) consists of an assignment P on N and a payoff vector $r \in \mathbb{R}^n$ satisfying that $r_i = v(\{i\})$ if agent $i \in N$ is an independent in P, and $r_j + r_h = v(\{j, h\})$ if agents $j \in N$ and $h \in N$ are partners of each other in P.

An allocation consists of an assignment and a payoff vector satisfying that the payoff of an independent is equal to his value and if two agents are matched they get a total payoff equal to their joint value. The following equilibrium concept yields for a partnership formation problem a stable set of allocations.

[1] See Lemma 4 which generalizes the fundamental Birkhoff–von Neumann theorem on doubly stochastic matrices. The latter theorem is the tool for analyzing assignment markets.