I study a market where agents with unit demand jointly own heterogeneous goods. In this market, the existence of an efficient, incentive compatible, individually rational, and budget balanced mechanism depends on the shares of the agents. I characterize the set of shares for which having such a mechanism is possible. This set includes the symmetric allocation and excludes the allocation in which every agent owns a separate good.

1. Introduction

How should a condominium association assign units to its member families after the construction is completed? In practice, a random order of families is drawn. According to the draw, each family then chooses from the remaining units until everyone has chosen a unit. This procedure is called the random serial dictatorship. Now, suppose that the second family is willing to pay a handsome subsidy to get the unit chosen by the first family. These two families could both benefit if they swapped their houses and the second family paid some money to the first family. Hence, the serial random dictatorship is not efficient. I characterize the set of initial shares for which having a mechanism with the required properties is possible. If further show that this set is convex. This result is similar to that of Cramton et al. (1987) (CGK) even though the setups are different. To be more explicit, in CGK there is one item jointly owned by many agents. In that setup, they characterize the set of initial shares when efficient partnership dissolution is possible and they provide a mechanism to dissolve the partnership. If there are more goods without any restrictions on how many goods agents own initially or finally, their analysis can still be used. In particular, if efficient partnership dissolution is possible, then their mechanism can be used good-by-good to dissolve all the partnerships. In contrast, the current model requires each agent to own one unit initially and finally. This is motivated by the aforementioned examples. Because of this difference, the results in CGK do not apply to the current problem.

A special case is when all agents have the same shares, for which I show the existence of a mechanism with the required properties. This is the case in the condominium association and the room assignment–rent division problem. Another special case of the setup is when each agent owns one good individually. This can be thought of as a housing market where people own one house but want to relocate. When \( n = 2 \) this case can be modified such that it coincides with the setup in Myerson and Satterthwaite (1983) who study a model with one buyer and one seller with a good. They show that if agents have overlapping values, then it is impossible to trade efficiently. I extend their classic impossibility result for general \( n \).
agents report their own types \( \theta_i \) to get \( p_i(\theta_i) \) share of the goods and a monetary transfer \( t_i(\theta_i; \alpha) \). Let \( P_i(\theta_i) = \mathbb{E}_{\alpha_i}[p_i(\theta_i)] \) be the expected shares that agent \( i \) is going to get from participation and \( T_i(\theta_i; \alpha) = \mathbb{E}_{\alpha_i}[t_i(\theta_i; \alpha)] \) be the expected transfers. Therefore, in the interim stage when agent \( i \) only knows \( \theta_i \) their expected gain from participating in the mechanism is \( U_i(\theta_i; \alpha) = \theta_i \cdot (P_i(\theta_i) - \alpha_i) + T_i(\theta_i; \alpha) \). If agent \( i \) misreports their type to be \( \theta_i' \) then their utility is \( \theta_i' \cdot (P_i(\theta_i') - \alpha_i) + T_i(\theta_i'; \alpha) \).

A mechanism \((p, t)\) implements allocation rule \( p \) if truth-telling is a Bayesian–Nash equilibrium. Such a mechanism is called (interim) incentive compatible (IIC) and satisfies the following inequality:

\[
U_i(\theta_i; \alpha) \geq \theta_i \cdot (P_i(\theta_i) - \alpha_i) + T_i(\theta_i; \alpha) \quad \text{for all } \theta_i \text{ and } \theta_i'.
\]

A mechanism \((p, t)\) which gives non-negative expected utility in the interim stage to all the agents is called (interim) individually rational (IRR): \( U_i(\theta_i; \alpha) \geq 0 \) for all \( \theta_i \). Furthermore, this mechanism is called ex-ante budget balanced (EABB) if the sum of expected transfers is zero, that is \( \mathbb{E}_\alpha[\sum_i t_i(\theta_i; \alpha)] = 0 \). Similarly, it is called (ex post) budget balanced (BB) if \( \sum_i t_i(\theta_i; \alpha) = 0 \) for all \( \theta_i \). It is efficient (EF) if \( \sum_i \theta_i \cdot p_i(\theta_i) \geq \sum_i \theta_i \cdot p_i(\theta) \) for all allocation rules \( p \) and for all \( \theta_i \). In this case \( p \) is also called the efficient allocation rule. Note that efficiency is a condition on the allocation rule and pins it down generically. Moreover, for generic \( \theta_i \), the efficient allocation rule assigns one separate good to each agent. Furthermore, the efficient allocation rule satisfies the following.

**Fact 1.** Suppose that \( p \) is the efficient allocation rule. Then \( p_i(\theta_i) = p_i(\theta_i + c1^\top) \) for all \( i \) and almost all \( \theta \) where \( c \in \mathbb{R}^n \).

This fact follows directly from the definition. If \( \sum \theta_i \cdot p_i(\theta_i) \geq \sum \theta_i \cdot p_i(\theta) \) for all \( p \), then \( \sum (\theta_i + c1^\top) \cdot p_i(\theta_i) \geq \sum (\theta_i + c1^\top) \cdot p_i(\theta) \) for all \( p \).

Since I concentrate on efficient mechanisms, the only choice variable is the transfer function.

### 3. The existence condition

In this section I provide a necessary and sufficient condition for a mechanism to satisfy the properties listed. First I state a revenue equivalence theorem below, from which it follows that any two EF and IIC mechanisms are interim payoff equivalent up to a constant. Therefore, to analyze the existence of such a mechanism which also satisfies EABB and IIR, it is sufficient to use a VCG mechanism which is EF and IIC. Hence, I derive a necessary and sufficient condition to have an EF, IIC, IIR, and EABB condition which is also implementable in dominant-strategies since VCG mechanisms make it a dominant strategy for agents to report their types truthfully.\(^4\) By using the transformation in Arrow (1979) and d’Aspremont and Gérard-Varet (1979) (Arrow–AGV transformation), EABB can be strengthened to BB at the expense of weakening dominant-strategy incentive compatibility to IIC.

#### 3.1. Some preliminary results

To begin, I give the revenue equivalence condition for the setup.

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\(^2\) Summations are always over \( i \) unless stated otherwise.

\(^3\) Given an allocation problem, \( \alpha \) is a constant which is commonly known by all the agents. Even though agents do not announce \( \alpha \) in the DRM, the payments can depend on it. To analyze the problem for different values of \( \alpha \), it is explicitly written in the transfer function.

\(^4\) Here, \( 1 \) is the \( n \)-dimensional vector consisting of ones and \( c1^\top \) is the matrix multiplication of \( c \) with the transpose of \( 1 \).

\(^5\) Dominant strategy incentive compatibility for a DRM requires that each agent prefers to report her type truthfully regardless of what other agents report.
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