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Chaotic behavior in manufacturing systems

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Abstract

In this article, we present a methodology derived from non-linear dynamic systems (NLDS) theory for analyzing the dynamic behavior of manufacturing systems. Some simple production systems are simulated, for which a chaotic behavior can be observed under certain dispatching rules and utilization levels. The dynamic behavior of a reactive system is studied; i.e., a system in which there is no previous schedule but jobs and operations are assigned to machines according to the state of the system. A discrete event model is used to represent the manufacturing system.

Keywords: Flexible assembly systems; Dynamic systems; Chaos

1. Introduction

In recent years, an increasing interest in chaotic behavior in physical systems has arisen. During the last decade, several publications in different scientific and technological domains describe chaotic phenomena. In the domain of production systems, some researchers have studied this problem, particularly Horns (1989), Tönshoff and Glöckner (1992), Massotte (1993), Deshmukh and Talavage (1995), Alfaro et al. (1997), and Rousseau (2000) among others. These authors show that chaotic behavior can occur in manufacturing systems. In this article, we study the dynamic behavior of a *reactive system*. That is, a system where there is no previously determined schedule and the assignment of operations to machines is done according to the state of the system. We have chosen discrete event simulation to represent the system in first place, and secondly, we analyze its behavior by using non-linear dynamic systems (NLDS) theory.

The article is divided into three sections: Section 2 presents the method used for analysis, which is based on Taken's theorem. Section 3 shows the method as applied to some simple manufacturing systems. Section 4 shows results for an actual FMS for assembly tasks. The results show that even in the case of simple systems under deterministic rules such as SPT (Shortest Processing Time), FIFO (First In First Out), or HPT (Highest Processing Time), a complex system's behavior

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can be obtained. For the real flexible manufacturing cell, several state and performance variables are analyzed by means of Fourier's spectrum and the Lyapounov's fractal dimension. It is shown that for the specific operating conditions, the cell's dynamics is chaotic.

2. Method of analysis

When dealing with NLDS analysis, a fundamental result is provided by Taken's theorem (Takens, 1981). This theorem states that if a system has a chaotic behavior then it is possible to obtain the number of variables and their values describing such a system by means of a time series of any variable from that system.

In order to apply this theorem to a manufacturing system, it is necessary that machines have a relationship. For this purpose, we define the following concepts:

(i) A manufacturing system can be regarded as one type of system with interacting machines and parts.
(ii) Relation $R(a,b)$: a machine a is related to a machine b when there exists a flow of parts from one to another, even if this flow was not direct.
(iii) Dominance relation $R(a \to b)$: a machine a is said to dominate a machine b when the change of a parameter of machine a modifies the dynamics of the queue at machine b.
(iv) Reciprocity relation $R(a \leftrightarrow b)$: a machine a is said to be reciprocal with a machine b when the change of a parameter in machine a modifies the dynamics of the queue in machine b, and vice versa.

In order to apply Taken's theorem, it is necessary to have a reciprocal relation between machines. Fig. 1 shows a system of eleven machines with three sub-systems or groups where the reciprocal relation holds (G1, G2, G3). Each of these sub-systems may have different types of behavior. However, it is not possible to apply the theorem to sub-systems of machines eight and nine.

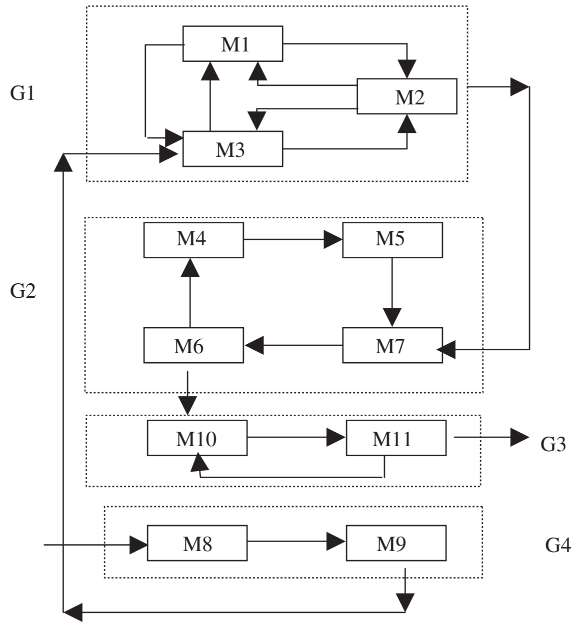

Fig. 1. Diagram of the system under study.

For the study of a manufacturing system there are basically two alternatives: an *analytical model* or a *simulation model.* In this work, we have chosen simulation modeling because it fits better to the system's characteristics. The simulation runs yield a number of useful variables such as: total flow time, time between departures, number in system, queue length, etc. In this work, we have chosen a *time persistent* variable which is the *mean number of units in queue.* Let this variable be $y(t)$. If we call $x(t)$ the discrete integer variable which gives the number of parts in a queue at time t, then $y(t)$ can be defined by

$$y(t) = \frac{\int_{n\,\Delta T}^{(n+1)\,\Delta T} x(\tau)\,\mathrm{d}\tau}{\Delta T},$$
$$t \in [n\,\Delta T, (n+1)\,\Delta T] \quad \text{for } n = 0, 1, 2... \tag{1}$$

Fig. 2 shows a plot of $x(t)$ (the time-persistent curve) and $y(t)$ (the continuous curve). In the example shown in Fig. 2, the size of ΔT has been obtained by simulations. The criteria for choosing the size of ΔT is that the number of sampled points of $y(t)$ is large enough to show its behavior. A too-large Δt value will lead to loss of information