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An approximation formula for basket option prices under local stochastic volatility with jumps: An application to commodity markets[☆]

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ABSTRACT

This paper develops a new approximation formula for pricing basket options in a local-stochastic volatility model with jumps. In particular, the model admits local volatility functions and jump components in not only the underlying asset price processes, but also the volatility processes. To the best of our knowledge, the proposed formula is the first one which achieves an analytical approximation for the basket option prices under this type of the models.

Moreover, in numerical experiments, we provide approximate prices for basket options on the WTI futures and Brent futures based on the parameters through calibration to the plain-vanilla option prices, and confirm the validity of our approximation formula.

1. Introduction

The basket options are one of the most popular exotic-type options in the commodity and equity markets. However, it is a tough task to calculate a basket option price with computational speed fast enough for practical purpose, mainly due to the difficulty of the analytical tractability and its high dimensionality. For instance, although the Monte Carlo method is easy to implement, it requires a substantial computational time to obtain an accurate value. Also, the numerical methods for the partial differential equations (PDEs) have been well developed, but it is still very difficult to solve high dimensional PDEs with accuracy and computational speed satisfactory enough in the financial business. To overcome the difficulties, this paper develops a new analytical approximation formula for basket options. In particular, to the best of our knowledge, our approximation formula is the first one which achieves a closed form approximation of basket options under stochastic volatility models with local volatility functions and jump components for not only the underlying asset price processes, but also the volatility processes.

There exist a large number of preceding studies on pricing basket options. In the Black–Scholes model, Brigo, Mercurio, Rapisarda and Scotti [1] applied a moment matching method to approximate basket option prices. Deelstra, Liinev and Vanmaele [2] derived the lower and upper bounds for basket call option prices with comonotonic approach.

Under a Local volatility model, Takahashi [3] showed an approximation for basket option prices with an asymptotic expansion technique. Bayer and Laurence [4] used a heat kernel expansion and the Laplace approximation method to derive very accurate approximate prices of basket options.

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In a local volatility jump–diffusion model, Xu and Zheng [5] derived a forward partial integral differential equation (PIDE) for basket option pricing and approximated its solution. Also, Xu and Zheng [6] applied the lower bound technique in [7] and the asymptotic expansion method in [8] to obtain the approximate value of the lower bound of European basket call prices. Moreover, when the local volatility function is time independent, they suggested to have a closed-form expression for their approximation.

Under a local stochastic volatility model, Shiraya and Takahashi [9] has developed a general pricing method for multi-asset cross currency options which include cross currency options, cross currency basket options and cross currency average options. They also demonstrated that the scheme is able to evaluate options with high dimensional state variables such as 200 dimensions, which is necessary for pricing basket options with 100 underlying assets under stochastic volatility environment. Moreover, in practice, fast calibration is necessary in the option markets relevant for the underlying assets and the currency, which was also achieved in the work.

Models within the class of the so called local stochastic volatility (LSV) model are mainly used in practice: for example, SABR [10], ZABR [11], CEV Heston (e.g. [12]) and Quadratic Heston models (e.g. [12]) are well known. Nonetheless, the LSV model is not always enough to fit to a volatility smile and term structure. Hence, some advanced researches investigated a local stochastic volatility with jump model. Among them, Eraker [13] found that the models with jump components in the underlying price and volatility processes showed better performance in fitting to option prices and the underlying price returns' data simultaneously in stock markets. Pagliarani and Pascucci [14] derived an analytical approximation of plain-vanilla option prices by applying the adjoint expansion method. However, to the best of our knowledge no works have derived an analytical approximation formula for the option prices under a model which admits a local volatility function and jumps both in the underlying asset price and its volatility processes. This paper develops a formula for pricing basket options under the setting by extending an asymptotic expansion approach. This closed form equation has an advantage in making use of the better calibration to the traded individual options whose underlying assets are included in a basket option's underlying.

In fact, our numerical experiments provide estimates of basket option prices based on the parameters obtained by calibration to the market prices of WTI futures options and Brent futures options. Then, those estimated prices are compared with the prices calculated by Monte Carlo simulations.

An asymptotic expansion approach in finance was initiated by Kunitomo and Takahashi [15], Yoshida [16], and Takahashi [17,3], which provides us a unified methodology for evaluation of prices and Greeks in general diffusion setting. Recently, the method was further developed to be applied to the forward backward stochastic differential equations (FBSDEs). (See [18–23] for the details.)

Although the method was extended to be applied to a jump–diffusion model by Kunitomo and Takahashi [24] and Takahashi [25,26], they concentrated on approximation of only bond prices or/and plain-vanilla option prices under a local volatility jump–diffusion model, and did not derive higher order expansions than the first order for the option pricing. Subsequently, Takahashi and Takehara [27] found a scheme for pricing plain-vanilla options in a jump–diffusion with stochastic volatility model. However, thanks to a linear structure of the underlying asset price process in their model they separated the jump component with a known characteristic function and then applied the expansion technique developed in the diffusion models. Hence, their scheme cannot be applied directly to more general models nor basket option pricing. The current work generalizes these preceding researches in the asymptotic expansion approach.

The organization of the paper is as follows: After the next section briefly describes our model for basket options, Section 3 derives a new approximate pricing formula, and Sections 4 and 5 show numerical examples. Particularly, Section 5 provide approximate prices for basket options on the WTI futures and Brent futures based on the parameters through calibration to the plain vanilla option prices. Section 6 concludes. Appendix shows the derivation of the coefficients in the pricing equation and the conditional expectation formulas necessary for obtaining the main theorem.

2. Model

This section shows the model of the underlying asset prices and their volatility processes, which is used for pricing the European type basket options.

In particular, suppose that the filtered probability space $(\Omega, \mathbf{F}, \mathbf{P}, \{\mathbf{F}_t\}_{t\geq 0})$ is given, where $\mathbf{P}$ is an equivalent martingale measure and the filtration satisfies the usual conditions. Then, $(S_t^i)_{t\in[0,T]}$ and $(\sigma_t^i)_{t\in[0,T]}$, $i = 1,\ldots,d$ represent the underlying asset prices and their volatilities for $t \in [0,T]$, respectively. Particularly, let us assume that S_T^i and σ_T^i are given by the solutions of the following stochastic integral equations:

$$S_T^i = s_0^i + \int_0^T \alpha^i S_{t-}^i dt + \int_0^T \phi_{S^i}\left(\sigma_{t-}^i, S_{t-}^i\right) dW_t^{S^i} + \sum_{l=1}^n \left(\sum_{j=1}^{N_{l,T}} h_{S^i,l,j} S^i_{\tau_{j,l}-} - \int_0^T \Lambda_l S_{t-}^i \mathbf{E}[h_{S^i,l,1}]dt \right), \tag{1}$$

$$\sigma_T^i = \sigma_0^i + \int_0^T \lambda^i(\theta^i - \sigma_{t-}^i)dt + \int_0^T \phi_{\sigma^i}\left(\sigma_{t-}^i\right) dW_t^{\sigma^i} + \sum_{l=1}^n \left(\sum_{j=1}^{N_{l,T}} h_{\sigma^i,l,j} \sigma^i_{\tau_{j,l}-} - \int_0^T \Lambda_l \sigma_{t-}^i \mathbf{E}[h_{\sigma^i,l,1}]dt \right), \tag{2}$$

where s_0^i and σ_0^i, $i = 1,\ldots,d$ are given as some constants. The notations are defined as follows: