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Journal of Economic Theory 157 (2015) 295–314

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Matching markets under (in)complete information ☆

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Received 18 February 2011; final version received 20 November 2012; accepted 14 October 2014

Available online 13 January 2015

Abstract

We introduce incomplete information to centralized many-to-one matching markets. This is important because in real life markets (i) any agent is uncertain about the other agents' true preferences and (ii) most entry-level matching is many-to-one (and not one-to-one). We show that given a common prior, a strategy profile is an ordinal Bayesian Nash equilibrium under incomplete information in a stable mechanism if and only if, for any true profile in the support of the common prior, the submitted profile is a Nash equilibrium under complete information in the direct preference revelation game induced by the stable mechanism.

JEL classification: C78; D81; J44

Keywords: Many-to-one matching market; Stability; Incomplete information

☆ We are grateful to two referees and an associate editor for their helpful comments and suggestions. L. Ehlers acknowledges financial support from the SSHRC (Canada) and the FQRSC (Québec). Support for the research of J. Massó was received through the prize "ICREA Acadèmia" for excellence in research, funded by the Generalitat de Catalunya. He also acknowledges the support of MOVE (where he is an affiliated researcher) and the Government of Catalonia, through grants SGR 2009-419 and 2014-515. His work is also supported by the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centers of Excellence in R&D (SEV-2011-0075) and FEDER grant ECO2008-04756 (Grupo Consolidado C). Part of this research was done while J. Massó was visiting the Université de Montréal and while L. Ehlers was visiting the Universitat Autònoma de Barcelona; the visits were financed by CIREQ and CREA, respectively.

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http://dx.doi.org/10.1016/j.jet.2015.01.008

1. Introduction

Centralized many-to-one matching markets operate as follows to match the agents from two sides, the firms (colleges, hospitals, schools, etc.) and the workers (students, medical interns, children, etc.): a centralized clearinghouse collects for each participant a ranked list of potential partners and matches via a mechanism firms and workers on the basis of the submitted ranked lists. In applications many of the successful mechanisms are stable.[1,2] The literature has considered stability of a matching (in the sense that all agents have to be matched to acceptable partners and no unmatched pair of a firm and a worker prefer each other rather than the proposed partners) to be its main characteristic in order to survive.[3] This is puzzling because there exists no stable mechanism which makes truth-telling a dominant strategy for all agents (Roth [24]). Therefore, an agent's (submitted) ranked lists of potential partners are not necessarily his true ones and the implemented matching may not be stable for the true profile. The literature has studied intensively Nash equilibria of direct preference revelation games induced by different stable mechanisms under complete information.[4]

We use the (ordinally) Bayesian approach in many-to-one matching markets by assuming that nature selects a preference profile according to a commonly known probability distribution on the set of profiles (a common prior).[5] Since matching markets require to report ranked lists and not their specific utility representations, we stick to the ordinal setting and assume that probability distributions are evaluated according to the first-order stochastic dominance criterion. Then, a strategy profile is an ordinal Bayesian Nash equilibrium (OBNE) if, for every von Neumann Morgenstern (vNM)-utility function of an agent's preference ordering (his type), the submitted ranked list maximizes his expected utility in the direct preference revelation game induced by the common prior and the mechanism.[6] For direct preference revelation games under incomplete information induced by a stable mechanism, our main result shows a link between Nash equilibria under complete information and OBNE under incomplete information. More precisely, Theorem 1 states that, given a common prior, a strategy profile is an OBNE under incomplete information in a stable mechanism if and only if for any profile in the support of the common prior, the submitted profile is a Nash equilibrium under complete information at the true profile in the direct preference revelation game induced by the stable mechanism.

[1] See Roth [25], Roth and Peranson [31], and Roth [30] for a careful description and analysis of the American entry-level medical market. Roth [29], Kesten [16], Ünver [36], and Ehlers [11] describe and analyze the equivalent UK markets.

[2] Chen and Sönmez [8], Ergin and Sönmez [13], and Abdulkadiroğlu, Che, and Yosuda [1] study the case of public schools in Boston, Abdulkadiroğlu and Sönmez [4] study the cases of public schools in Boston, Lee County (Florida), Minneapolis, and Seattle, and Abdulkadiroğlu, Pathak, and Roth [2,3] study the case of public high schools in New York City.

[3] See, for instance, Roth [25] and Niederle and Roth [21].

[4] See Dubins and Freedman [10], Roth [24,26,27], Gale and Sotomayor [15], Shin and Suh [34], Sönmez [35], Ma [17,18], and Alcalde [6].

[5] Roth [28] is the first paper studying strategic incentives generated by stable mechanisms under incomplete information. He shows that for particular vNM-utility representations of the ordinal preferences, Bayesian Nash equilibria under incomplete information may not satisfy appealing properties of Nash equilibria under complete information. Chakraborty, Citanna, and Ostrovsky [7] study two-sided matching markets with interdependent preferences.

[6] This notion was introduced by d'Aspremont and Peleg [5] who call it "ordinal Bayesian incentive-compatibility". Majumdar and Sen [20] use it to relax strategy-proofness in the Gibbard–Satterthwaite Theorem. Majumdar [19], Pais [22], and Ehlers and Massó [12] have already used this ordinal equilibrium notion in one-to-one matching markets.