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# Conditional Sharpe Ratios

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ABSTRACT

Facing investment choices, investors may care more about potentially excess losses in a downtrend market than excess gains in an upside market. *Conditional Sharpe ratios* (*CSR*) are statistical ordinates of *conditional stochastic dominance* (*CSD*) that measure lower partial risk-adjusted excess returns of an asset with respect to return distribution on the benchmark. A multiple comparison of serial *CSR* statistics thus provides an overall view of portfolio performance corresponding to different market scenarios. An example demonstrates that *CSR* is able to discriminate funds' downside performance which the conventional Sharpe ratio generally fails to do. A large out-of-sample analysis of US mutual fund shows that *CSR* has predictability for portfolio future performance.


## 1. Introduction

Portfolio selection and performance evaluation focuses on excess return relative to the benchmark that is generated by managers' superior skills of security analysis and effective asset allocation and reallocation. The excess returns are simply the differential returns between the portfolio and its benchmark. The widely accepted tenet that investors are risk-averse leads to risk-adjusted performance measures which characterize the trade-off between risk and return. Based on restrictions on individuals' preference, and/or an assumption of symmetric return distribution on assets, valid

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performance measures are derived under the mean–variance theory. Sharpe (1994) suggested a simple *relative reward-to-variability ratio* that can be used to measure portfolio performance. This expression of Sharpe's ratio, defined as the differential return between a fund and the benchmark, divided by the standard deviation of the differential returns, is a *generalized* version as the conventional *Sharpe Ratio* (1966 and 1975). Alternatively, if the return generating process of assets is linear in form with respect to common market factors, then the expression of *relative reward-to-variability* ratio is *alpha* (the constant term of a estimating regression) over *omega* (the residual standard deviation). This approach is called as the information ratio (*IR*) which is also characterized by a normalized excess return by additional units of residual risk.[2]

The Sharpe ratio or *IR* is developed from assumptions that the return distribution of assets is symmetrical, and individuals' utility function of wealth is strictly concave. However, these assumptions have recently been challenged based on empirical evidence as well as observed behavior. One important observation is that when individual investors face a choice under uncertainty, an asset is unattractive to hold if the asset tends to move downward in a declining market more than it moves upward in a rising market. Therefore, to hold assets that strongly co-vary with a downward market, a premium or additional expected excess return will be required by investors who are sensitive to downside losses relative to upside gains. This being the case, the unattractive asset may not be discriminated by the traditional performance measures of the Sharpe ratio in that the overall risk-adjusted return could remain unchanged. To discriminate unfavorable alternatives, the selection criteria thus need to have two important characteristics: the ability to measure simultaneously investment performance under different market conditions, and the ability to consider the joint nature of investment outcomes with the market distribution. This suggests that a single measure alone is unable to accomplish the goal; a multiple comparison approach could be an alternative solution.

Numerous studies of empirical distributions of returns on financial assets reject the symmetry assumption in favor of a skewed and fat-tailed distribution. Some evidence appears in Mandelbrot (1963), Fama (1965), Officer (1972), Gray and French (1990), and Focardi and Fabozzi (2003). The rationale for this rejection is that return profiles show losses that are large and too frequent to fit into a balanced symmetrical distribution. Mean–variance theory assumes individuals are strictly risk-averse with respect to wealth. However, increasing arguments from a behavioral prospective suggests that investors have different attitudes toward risk between gain and loss. Ang et al. (2006) empirically find that investors are much more sensitive to downside losses relative to upside gains.[3] An approximate 6% annual downside risk premium is required by investors to hold stocks that have strong co-movement with a declining market. Consequently, an asset is unattractive or underperforming, if it has excess loss in a down-market more than excess gain in an up-market. At the same time, Post and levy (2005) applied stochastic dominance efficiency to equity market returns and found that the Markowitz's reverse S-shape utility function with risk-aversion to loss and risk-seeking for gains describes the return pattern on stocks. They conclude that investors have twin desires for downside protection in bear markets and for upside potential in bull market. Although there is an ongoing debate about what form or shape of utility function fits individual risk behavior, it is recognized that the traditional assumption of strict risk-aversion fails to be supported by empirical evidence. Specifically, individuals' risk-aversion could vary as market conditions change; investors seem more concerned about downside loss than upside gain.

[2] In a classical seminal paper, Treynor and Black (1973) theoretically proved that under a single-factor market model, the alpha–omega ratio contains information about not only the superiority of manager's security analysis but his/her ability to exploit the superior information in effective asset reallocation. The value-added return of a portfolio under the same risk of the benchmark (the $M^2$ measure) can be calculated from the IR. The $M^2$ measure is equal to the Sharpe ratio multiplied by the standard deviation of the benchmark index and then added to the risk-free rate. The squared value of the active portfolio's Sharpe ratio equals that of the benchmark's Sharpe ratio plus the squared value of *IR*. In a multifactor framework, Grinold and Kahn (1999) decomposed the information ratio and proposed a "fundamental law of active management". Specifically, IR intuitively contains two components of important information: the breath which is the number of distinct signals for measuring managers' skills of securities selection and the information coefficient that is the average of those signs for measuring managers' ability of forecasting the correct alpha, respectively.

[3] Theoretically, it has been long recognized that risk-averters care differently about downside losses versus upside gain (see Roy (1952) and Markowitz (1952)).