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Conflict Management: Apollonius in airspace design

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ABSTRACT

Conflict detection is the process by which potential collisions between aircraft are identified. Developing efficient detection methods is an engineering priority since the prevention of a mid-air collision is a major safety constraint for all airspace design. Proximity between two aircraft each flying with instantaneous uniform straight motion is re-examined. The Apollonius theorem for constructing a circle shows that for a constant speed ratio the locus of all collision points is a circle in the 2D conflict plane (generalised to a sphere in 3D space). This circle provides an effective method and reference for determining the set of all 3D non-allowed steering directions (NASD), which, if followed by ownaircraft, would result in either a collision or proximity of operational concern. The NASD paradigm permits the identification of a protection zone encapsulating the region of potential collision. The method has been applied in the design of static airspace structures and accident analysis as well as providing an explicit solution for the determination of a surface of singularities that are characteristic of force-field guidance laws. The NASD paradigm provides deeper insight into the conflict detection process and represents an increase in the utility, diversity and assessment of design options for future systems.

1. Introduction

1.1. Conflict management in air transport

A new mode for a flight management system is presented that has applications in aircraft-to-aircraft conflict detection and management (see, Rand and Eby, 2004; ICAO, 2005). The focus of the paper is on the mathematical modelling and analysis required in the conceptual engineering design of the mode. The safety critical operational imperative means that the mode must be dependable (safe, reliable and having integrity). The results of this paper support the further development of these concepts.

During flight the crew must be able to predict both the future state between their aircraft and any other proximate aircraft (an intruder), and whether, at the Point of Closest Approach (PCA) between the aircraft, the relative range would be reduced to below certain prescribed minima (Xu and Rantanen, 2007). It is for these close aircraft-to-aircraft encounters that interaction is considered. One of the design questions arising is:

> *"What is the best available proximity information that can be presented to the pilot given that a 3D position and a 3D velocity vector can be exchanged between aircraft?"*

Answering this question reveals the essence of the design requirements for conflict detection. Our investigations over the past decade have included aircraft guidance and intercept, integration of Unmanned Aerial Vehicle operations into civil airspace, airspace allocation design and port management in a maritime setting. All have shown that there is a broad application for the geometric theorem known as the Circle of Apollonius (attributed to Apollonius of Perga, circa 262 – 190 B.C.E.) (Coxeter, 1989). This theorem can be applied, in engineering design and in actual operations, to determine the locus for all possible points in space for which a zero miss-distance between flight-paths would occur.

A rigorous analytic extension and characterisation of the geometric theorem is presented with an introduction to the Non-allowed Steering Direction (NASD) paradigm. Together these lead to a refinement of the Point of Closest Approach paradigm. A demonstration of the diversity of applications that would benefit from this research is also presented.

Two different perspectives further illuminate the relevance of this research. First, in uncontrolled airspace flying operations are often conducted at lesser physical margins than is experienced by operations in controlled airspace where spacing between aircraft is managed by reference to either a spatial or temporal separation standard. Uncontrolled airspace operations are normative in rural and remote areas and these areas are not insignificant on a continental basis. For example: 99.8% of Canada was designated rural in 2006 (Bråthen and Halpern, 2012) and 99.2% of continental

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Nomenclature

$\|\|\cdot\|\|$	norm of the vector: $\cdot$
$\underline{A}$, $\|\|\underline{A}\|\|$	kinematic vector, norm of the vector
$\overline{AB}$, $\|\overline{AB}\|$	geometric segment, modulus of the segment (NB: AB will be used in place of $\|\overline{AB}\|$ to simplify equations)
$\underline{AB}$, $\|\|\underline{AB}\|\|$	geometric segment – treated as a vector, norm of the vector
k	speed ratio, $k \geqslant 1$
k^+, k^-	maximum, minimum value of k in a practical design implementation
t	time (s)
ΔT	time differential: two aircraft passing through a common waypoint (s)
ΔD	distance differential: two aircraft passing through a common waypoint
ΔV_{io}	variation in speed with respect to the operational point, (V_{1o}, V_{2o}); $i = 1, 2$
x_S	x co-ordinate, $S \in \{I, E, H, C, T\}$ where set members denote cardinal points
y_S	y co-ordinate, $S \in \{I, E, H, C, T\}$ where set members denote cardinal points
$(\underline{x}, \underline{y})$	Cartesian Coordinate unit vectors where $\underline{x} \equiv \underline{U}_{LOS}$
O	origin or coordinate system, position of ownaircraft (aka pursuer, ownship)
E	position of the intruder (aka evader, or threat) aircraft
$\underline{P}_1$, $\underline{P}_2$	position vector of ownaircraft, intruder respectively
R	radius of the Apollonius Circle
$\underline{R}_{LOS}$, $\underline{U}_{LOS}$	relative position (aka Line of Sight (LOS)) vector, unit vector
$\underline{R}_{LOS}$	relative range (aka line of sight range, $R_{LOS} = x_E$)
$\underline{V}_1$, $\underline{V}_2$	velocity vector for ownaircraft, intruder respectively
$\underline{V}_{1y}$, $\underline{V}_{2y}$	component of ownaircraft, intruder velocity vector orthogonal to the LOS
$\underline{V}_R$, $\underline{U}_{VR}$	relative velocity vector, unit vector
$\underline{V}_1^+, \underline{V}_1^-$	ownaircraft velocity vectors for $\pm\eta$ to $\underline{U}_{LOS}$ respectively
$\underline{V}_{21}^+, \underline{V}_{21}^-$	intruder velocity vectors for head-on $\underline{V}_0^+, \underline{V}_0^-$ cases respectively
$\overline{V}_{22}^+, \overline{V}_{22}^-$	intruder velocity vectors for in-trail $\underline{V}_0^+, \underline{V}_0^-$ cases respectively
(V_{1o}, V_{2o})	instantaneous nominal operational point in the domain $V_1 \times V_2 \in \mathbb{R}^2$
$W(x, y)$	general waypoint relative to O with coordinates (x, y)
α	aspect angle for intruder relative to LOS vector
η	Collision direction for ownaircraft relative to LOS vector
$\theta, \psi, \sigma, \rho, \gamma$	general angles of the Apollonius construct
$\underline{\Gamma}$	displacement vector: Apollonius Circle centre (C) from the intruder (E)
$\breve{\underline{\Gamma}}$	iso-orthogonal vector for $\underline{\Gamma}$: that is, iso-norm and orthogonal to $\underline{\Gamma}$
$x(y)z$	contour sequence: numbers starting with x, incrementing by y, until z

Australia is Class G airspace from ground level to the flight level, FL185. In these regions it is routine for crew to experience aircraft proximity at less than 0.1 NM and less than 200 s to PCA, particularly in the circuit area.

Second, from the international engineering perspective, conflict detection is considered a part of conflict management, one of the seven principal functions of the Global Air Traffic Management Operational Concept (GATMOC). This project was established by the International Civil Aviation Organization (ICAO) to progress harmonisation of air traffic management systems on an international scale (ICAO, 2005). In support of this initiative there are international programs focused on the redesign of cockpit displays – see, e.g., research on the Cockpit Display of Traffic Information (CDTI), the Traffic Alert and Collision Avoidance System (TCAS) (Cleveland et al., 2011) and the Ecological Interface Designs (Ellerbrock et al., 2011). Each, in part, are motivated by the provision and design of new conflict detection cues. Such consideration also arises in support of proposed operational modes such as "full self-separation" envisaged in continuous descent arrivals in controlled airspace (ICAO, 2005).

Under the GATMOC architecture, the conflict management function is to be implemented hierarchically with air traffic flow optimisation ranging from continental scales, to regional flows in the presence of weather cells such as thunderstorms, through to the interaction of pairwise flight trajectories. Major traffic flows will be optimised in a process that removes conflicts from the predicted trajectories many months before an actual flight. When strategic optimisers no longer have sufficient time to run then tactical planners will provide adaption of the strategic solution for any local conflicts emerging due to short-term system disturbances and uncertainties such as adverse weather or changed flows. An "operational" status is deemed to exist when there remains insufficient time to wait even for tactical planners. While significant research has been conducted in planning traffic flows at the continental and sector levels the very local (large) scale aircraft-to-aircraft interaction remains underdeveloped. This research seeks to address some of the concerns for this lower echelon of the conflict management hierarchy.

1.2. Maritime transport

Although this paper has been written from the air transport perspective the research has relevance to other forms of transport, particularly maritime operations (being the original setting for its use). Chauvin and Lardjane (2008) investigated the decisions regarding conflict detection and resolution made by a watch officer onboard a ferry crossing the Dover Strait. They noted that over 400 vessels per day use the two main channels in the Strait while, of order, 70 ferries and 240 other vehicles pass across the channels. In particular they note that different interpretations of the regulations for resolving conflict exist and that these differences generate uncertainty regarding the resulting actions by vessels. The research of this paper provides a means to better visualise the true conflict situation thus minimising any resultant uncertainty and improving the possibility of a safe passage.

1.3. Presentation

In Section 2 the operational and engineering importance of the role that the Apollonius Circle plays in the engineering specification of Conflict Detection (CD) is discussed. The central role in the safety critical design imperative for Conflict Management (CM) functions is also discussed here.

A précis of the geometry and construction of the circle is then presented in Section 3. The spatial characteristics, analytic extensions and new interpretations of this classic theorem are provided in Section 4; temporal characteristics in Section 5. Section 6 sets deterministic bounds on the Non-Allowable Steering Direction (NASD) for both worstcase and bi-normal error characteristics. Section 7 illustrates applications in CD and Conflict Resolution (CR)