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Optimal allocation and consumption with guaranteed minimum death benefits, external income and term life insurance

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HIGHLIGHTS

- We examine the choices of CRRA individuals who own Variable Annuity with GMDB.
- We find joint effects of human capital and GMDB on CRRA individuals.
- We find that term life insurance can be considered as a substitute for GMDB.
- We find that fairly priced GMDB options fail to add value to a VA contract if a term life policy is available.

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ABSTRACT

Because human capital is often the largest asset an investor possesses when he is young, protecting human capital from potential risks should be considered as a part of overall investment advice. The risk of the loss of the policyholder's human capital – the mortality risk – to the household can be partially hedged by a term life insurance policy. Guaranteed Minimum Death Benefits (GMDB) in Variable Annuities (VA) can also help policyholders hedge the risk of the loss of human capital. Therefore, GMDB options and term life insurance can be considered as substitute goods. However, they are not perfect substitute as GMDB and term life have their own properties: Term life insurance has no correlation with equity markets, and it is purely regarded as a protection for human capital; the variable annuity products follow the performance of equity markets, and the GMDB is a protection against downside risks on equity markets as well as human capital. We find that fairly priced GMDB options fail to add value to a VA contract if a term life policy is available.

1. Introduction

Guaranteed Minimum Death Benefits (GMDB) in Variable Annuities (VA) have been drawing a lot of interest recently. Variable Annuities have historically been used as an accumulation vehicle to provide for retirement. In recent times, various guarantees have been added to these accounts, including Guaranteed Minimum Death Benefits (GMDB), which promise more than just return of the account value on death. In this paper, we examine the suitability of Return of Premium GMDB options, which are similar to European put options with a random maturity date corresponding to the time of the individual's death.

Financial professionals often argue that GMDBs are, at best, redundant and at worst, a poor investment. After all, term insurance is available at reasonable prices and it is therefore possible to buy a variable annuity without a rider and its associated fees, and a separate term life policy to protect the beneficiary. On the other hand, it is not possible to fully replicate the GMDB payoff with term insurance, as the GMDB pays off more when assets are low, whereas the term insurance payoff is insensitive to asset levels. If the beneficiary is risk averse, he may prefer a GMDB to term life insurance in some instances. In this paper, we examine this question.

In this paper, we assume that an individual owns a variable annuity contract (either with or without a GMDB rider and its associated fees) and makes decisions optimally in order to maximize the expected utility of lifetime consumption. The insured gets utility

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from consumption and has bequest motives. We include the effect on asset allocation from dissavings (consumption). We reflect bequest motives by including the utility of the recipient of the policyholder's guaranteed death benefits. The policyholder maximizes the discounted expected utility of the policyholders and beneficiaries by investing dynamically in an underlying fixed account and variable fund and withdrawing optimally.

We also assume the individual wishes to protect his beneficiary from loss of human capital. We assume labor income can be consumed in addition to the variable annuity withdrawals. Human capital is the present value of individual's remaining lifetime labor income, and it will influence individual's asset allocation choices, consumption choices and insurance choices. "The risks you can afford to take depend on your total financial situation, including the types and sources of your income exclusive of investment income" (see Malkiel, 2004, pg. 342). Hanna and Chen (1997) study optimal asset allocation by considering human capital. They conclude that the investors who have long investment horizons should apply an all equity portfolio strategy. Bodie et al. (1992) study investment strategy given labor income. They find that younger investors should put more money in risky assets than should older investors. Chen et al. (2006) take human capital into account, and argue that human capital affects asset allocation. There are roughly three stages of a person's life:[2] the first stage is the growing up and getting educated stage; the second stage is the accumulation stage, in which people work and accumulate wealth; the third stage is the retirement/payout stage. Human capital generates significant amount of earnings during the accumulation stage. As individuals save and invest, human capital is transferred to financial capital. Chen et al. (2006) provide an approach to making the individuals financial decisions in purchasing life insurance, purchasing annuity products and allocating assets between stocks and bonds.

Finally, the policyholder optimizes the combined utility through payment of premiums for term life insurance to protect his income stream. We investigate if the guarantee options add value to the contract even if the term life policy is available. Many papers study life insurance demand. Campbell (1980) derives solutions to optimal life insurance demand on mortality risk. His model introduces an insurance market to hedge the mortality risk and allows for the possibility that future tastes may be state-dependent. In his work, mathematical demand-for-insurance equations were derived to explicitly describe household's optimal responses to human capital uncertainty. Grace and Lin (2007) examine the life cycle demand for different types of life insurance by using the Survey of Consumer Finances. They find a relationship between financial vulnerability and term life insurance demand, and that older people demand less term life insurance. There are also a number of papers studying the joint demand of term life insurance and annuities. Hong and Rios-Rull (2007) construct an overlapping-generations model to analyze social security, life insurance and annuities for households. It reveals that the existence of life insurance opportunities for people is important in welfare terms. Purcal and Piggott (2008) use an optimizing lifetime financial planning model to explore optimal life insurance purchase and annuity choices. Their model incorporates the consumption and bequests in an individual's utility function. Policyholders' needs for life insurance and annuities varied across different levels of risk aversion and different bequest motives.

The effect of individuals maximizing their lifetime utility including VA riders have been analyzed previously in Bauer and Moenig (2011), Gao and Ulm (2012) and Steinorth and Mitchell (2012). However, these papers do not include the possibility of term life insurance purchases.

[2] This paper focuses on the accumulation stage.

We add to this literature by deriving the insured's optimal decisions in purchasing the term life policy, and allocating and withdrawing assets in his VA account. We price the GMDB from the insurer's perspective by incorporating the insured's choices in a risk neutral model. We also find that, when term insurance is available at reasonable prices, policyholders have a lower utility when their VA contains a fairly priced GMDB than when it does not.

2. The model

We assume that the insured and his beneficiary are risk averse with the same utility function. We apply a constant relative risk aversion (CRRA) type utility which has a functional form

$$u(c) = \begin{cases} \dfrac{c^{1-\gamma}}{1-\gamma}, & \gamma > 0, \gamma \neq 1, \\ \ln(c), & \gamma = 1. \end{cases}$$

An individual gets utility from his consumption c. γ is the coefficient of relative risk aversion, and the reciprocal of γ measures the willingness to substitute consumption between different periods. The CRRA assumption is chosen to produce the scaling relation in Eq. (12), which simplifies the numerical analysis. There is nothing otherwise special about CRRA utility and other utility functions could be analyzed in this framework.

2.1. External income and GMDB

Let us first assume that this individual purchases a variable annuity contract with GMDB options and makes a lump sum deposit to the variable annuity account. Once the insured receives labor income at the beginning of period t, he will make his consumption decision. If his labor income is not enough to support his consumption, he will make a decision to withdraw. Simultaneously, the GMDB level will be reduced proportionally with the withdrawal ratio. In our model, there are no periodic deposits, which means all the periodic income will be consumed at the current time t. After the consumption and withdrawal decision, still at time t, the policyholder will decide the allocation between fixed and variable subaccounts in the VA account. If the policyholder dies at time t, the amount in the VA account, which is protected by the GMDB, will be inherited by his beneficiary. The policyholder and beneficiary get utility through consumption. The beneficiary gets the bequest and maximizes her own utility by optimal allocation and withdrawal. The policyholder makes all these decisions to maximize the joint utility of his beneficiary and of himself.

If the insured has labor income, the objective function is

$$\max_{\omega_t, c_t} \sum_{t=1}^{T} \beta^t \left(\prod_{i=1}^{t} \phi_i \right) u(c_t) + \beta^T \left(\prod_{i=1}^{T} \phi_i \right) V_{T+1}(a_{T+1}) + \sum_{t=1}^{T} \beta^t \left(\prod_{i=1}^{t-1} \phi_i \right) (1-\phi_t) \zeta v_B(b_t). \quad (1)$$

The insured retires at the end of time T. ω is the percentage of wealth held in the variable subaccount and $1-\omega$ is the proportion of wealth allocated in the fixed rate subaccount. To be more realistic, we assume $\omega \in [0, 1]$, which means that there are no short sales. β is the subjective discount factor. ϕ is the survival rate. ζ denotes the strength of the bequest motive[3] and ranges from 0 to 1. If $\zeta = 0$, the insured has no bequest motive and leaves nothing to his beneficiary; if $\zeta = 1$, the insured is assumed to have the strongest bequest motive and will treat his beneficiary like himself. V is the policyholder's value function and v is the beneficiary's value function. If the insured dies before retirement, the beneficiary will get the larger of the account value or the guaranteed amount. Now we

[3] Arrondel et al. (1997) and Masson and Pestieau (1997) provide an overview of bequest motives. De Nardi (2004) and Ameriks et al. (2011) examine quantitative estimates of the bequest effect.