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A new type-2 fuzzy set of linguistic variables for the fuzzy analytic hierarchy process

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A B S T R A C T

The fuzzy analytic hierarchy process (FAHP) has been used to solve various multi-criteria decision-making problems where trapezoidal type-1 fuzzy sets are utilized in defining decision-makers' linguistic judgment. Previous theories have suggested that interval type-2 fuzzy sets (IT2 FS) can offer an alternative that can handle vagueness and uncertainty. This paper proposes a new FAHP characterized by IT2 FS for linguistic variables. Differently from the typical FAHP, which directly utilizes trapezoidal type-1 fuzzy numbers, this method introduces IT2 FS to enhance judgment in the fuzzy decision-making environment. This new model includes linguistic variables in IT2 FS and a rank value method for normalizing upper and lower memberships of IT2 FS. The proposed model is illustrated by a numerical example of work safety evaluation. Comparable results are also presented to check the feasibility of the proposed method. It is shown that the ranking order of the proposed method is consistent with the other two methods despite difference in weight priorities.

1. Introduction

The multi-criteria decision-making (MCDM) method is one of the most widely used approaches for evaluating typically multiple conflicting criteria. This approach often requires the decision-makers (DMs) to provide qualitative and quantitative measurements for determining the performance of each alternative with respect to criteria and the relative importance of evaluation criteria with respect to overall judgments (Abdullah, Sunadia, & Imran, 2009). Various authors have used MCDM methods naturally applied with fuzzy sets to deal with uncertainty alternative selection and overcome the vagueness limitations of MCDM methods (Cakir & Canbolat, 2008). Wang and Chang (2007) presented an application of the technique for order performance by similarity to ideal solution (TOPSIS) in evaluating initial aircraft training in a fuzzy environment. Another related research area of MCDM is the analytic hierarchy process (AHP) where pairwise comparisons have been applied to the task of organizing and analyzing complex decisions. AHP has been successfully applied in MCDM problems (Peng, Kou, Wang, & Wu, 2011) as one of the most frequent methods used. The main advantage of the AHP is its inherent ability to handle intangibles and less cumbersome mathematical calculations; it is also more easily comprehended than other MCDM methods. Fuzzy AHP (FAHP) was introduced to handle uncertainty in linguistic judgment. Initial research on FAHP was conducted by Laarhoven and Pedrycz (1983), who discussed the use of triangular fuzzy numbers and Lootsma's logarithmic least squares method to derive fuzzy weights and fuzzy performance scales. Buckley (1985) described FAHP as a direct extension of AHP, using trapezoidal numbers and the geometric mean method to derive fuzzy weights and fuzzy performance scores. The fuzzy number was introduced to express linguistic variables.

FAHP has been successfully applied in diverse applications. Cebeci (2009), for example, applied FAHP to compare enterprise resource planning system in a textile manufacturing company. Celik, Er, and Ozok (2009) investigated shipping registry selection using FAHP. Büyüközkan, Çifçi, and Güleryüz (2011) used FAHP to evaluate the service quality of the healthcare sector. Very recently, Lupo (2013a) and Lupo (2013b) proposed the use of FAHP and ServQual service performance in public transit service and the higher education sector. Most of the existing FAHP methods are built from linguistic variables based on type-1 fuzzy sets (T1 FS). With the development of type-2 fuzzy sets (T2 FS) and the concept of interval type-2 fuzzy sets (IT2 FS), decisions should receive more comprehensive evaluation thanks to the flexibility of spaces representing uncertainties than they do with T1 FS. T2 FS are characterized by a fuzzy membership function, as each element of this set is a fuzzy set in [0,1], unlike a T1 FS where the membership grade is a crisp number in [0,1] (Mendel, 2001). The membership functions of T2 FS are three-dimensional and include a footprint of uncertainty which is the new third dimension of T2 FS and provides additional degrees of freedom for directly modeling and

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handling uncertainties (Mendel, 2007; Trksen, 2002). Currently, interval type-2 fuzzy sets (IT2 FS) are widely used and have been successfully applied in perceptual computing (Mendel & Wu, 2010; Mendel et al., 2010), control systems (Hagras, Doctor, Callaghan, & Lopez, 2007; Jammeh, Fleury, Wagner, Hagras, & Ghanbari, 2009; Wagner & Hagras, 2010; Wu & Mendel, 2011). Related research is the application of T2 FS in decision-making. Ozen and Garibaldi (2004) used the shape of type-2 fuzzy membership functions to model variation in human decision-making. Wu and Mendel (2007) presented a method using the linguistic weighted average and IT2 FS for handling fuzzy multiple criteria hierarchical problems. Wu and Mendel (2008) proposed a fuzzy multiple criteria hierarchical method to make decisions by aggregating the opinions of DMs. Chen and Lee (2010) presented a new method for handling fuzzy multiple attribute group decision-making based on ranking values and the arithmetical operations of IT2 FS. Recently, Chen, Yang, Lee, and Yang (2012) presented a ranking IT2 FS method to handle fuzzy multiple attribute group decision-making problems. Wang, Liu, and Qin (2012) investigated the multi-attribute group decision-making problem in an IT2 fuzzy environment and developed an approach to handling the situation where the attribute values are characterized by IT2 FS and the information about attribute weights is partially known.

Developments in FAHP and IT2 FS in MCDM motivated us to explore the possible merger of these two entities. Our proposed work can be seen as an FAHP framework where fuzzy numbers are used in linguistic scales. The method proposed in this paper is basically similar to FAHP, but in the proposed method fuzzy numbers are given in IT2 FS. The insertion of IT2 FS in FAHP gives a new look to the FAHP framework. Among the implications of this are new linguistic variables, new reciprocal fuzzy numbers and the usage of rank values.

Most of the linguistic variables used in fuzzy decision-making approaches utilize knowledge of T1 FS. Wu, Tzeng, and Chen (2009), for example, used six scale linguistic variables of type-1 trapezoidal fuzzy numbers. The same linguistic variables are also used by Im and Cho (2013), Wang et al. (2014) and Cho and Lee (2013) in other applications of FAHP. Inspired by the homogeneous linguistic variable used by many authors and aiming to generalize the standard scale of fuzzy decision-making in interval value fuzzy numbers, this paper proposes a new linguistic variable using IT2FS purely for FAHP. This proposed approach allows each pairwise comparison matrix is made with trapezoidal IT2 FS as measurement scale instead of using classical trapezoidal fuzzy numbers to represent the judgment scales and the weights of criteria. Unlike FAHP, we focus on creating nine new linguistic scales rather than six linguistic scales. Therefore, our approach is more geared toward defining new linguistic scales in IT2 FS.

Furthermore, we rigorously define rank values to obtain a weighted DM matrix. Another contrast between this work and the FAHP method is that we used the concept of ranking trapezoidal IT2 FS proposed by Xu (2001) to normalize two different matrices consisting of upper and lower trapezoidal IT2 FS. This means our proposed method fits the definition of T2 FS. Upper and lower fuzzy numbers are now being normalized to single rank to make a seamless match with the FAHP framework. Compared with FAHP, this method offers a more detailed procedure and normalizes upper and lower numbers of IT2 FS by incorporating rank values in TOPSIS. The third contrast between the proposed method and the FAHP method is the new reciprocal fuzzy number. The reciprocal comparison concept of the decision elements is determined with IT2 FS. The concept of aggregation and the normalization process of the DM's matrices is calculated to obtain weighted ranking values for the matrices. The weighted ranking matrices are explained by the lower and upper theory of IT2 FS. Thus, it is more flexible and efficient to use an interval type-2 fuzzy set for expressing a linguistic evaluation because IT2 FS provides more flexibility in presenting uncertainties (Chen & Wang, 2013; Zhang & Zhang, 2013).

The proposed method could be applied to many practical applications of MCDM. The proposed preference scale may offer more comprehensive judgment due to the property of IT2 FS which can deal with more room of uncertainty. Furthermore, the interval type-2 fuzzy sets can be used as indicator of uncertainty whereas the larger length of the interval, the more uncertainty involves especially during the process of information gathering from the decision makers (Wu & Mendel, 2007). Another related step besides the linguistic scales is the reciprocal trapezoidal number which aims to associate with pairwise comparison of AHP. A new reciprocal fuzzy number in IT2 FS is introduced rather than the reciprocal TI FS (Zheng, Zhu, Tian, Chen, & Sun, 2012). Another unique trait in this proposed method is that we try to define a new upper–lower T2 FS pairwise comparison rather than one layer pairwise comparison (Zheng et al., 2012). By applying the reciprocals comparison concept, the relative importance of the decision elements can be determined through IT2 FS concept. This paper also extends the proposed method to evaluating work safety and early warning rating of hot and humid environments. This new method is a feasible procedure which applies the trapezoidal fuzzy number to IT2 FS as a preference scale. Despite the introduction of IT2 FS in defining the linguistic scale and the knowledge of TOPSIS rank values in normalizing upper and lower fuzzy numbers, the proposed method is made without loss of generality of Saaty's AHP procedures. The rest of this paper is organized as follows. In Section 2, we describe the basic concepts of trapezoidal fuzzy numbers and IT2 FS in the MCDM method. The proposed method of the IT2 FAHP framework is developed in Section 3. In Section 4, an illustrative example is presented to demonstrate the feasibility and consistency of the new IT2 FAHP. Section 5 concludes.

2. Preliminaries

This section introduces the basic definitions relating to trapezoidal fuzzy numbers, basic concepts of IT2 FS and arithmetic operations between trapezoidal IT2 FS.

2.1. Trapezoidal fuzzy numbers

A trapezoidal fuzzy number can be defined as $\tilde{m} = (a, b, c, d)$ where the membership functions $\mu_{\tilde{m}}$ of $\tilde{m}$ is given by:

$$\mu_{\tilde{m}} = \begin{cases} \frac{x-a}{b-a} & (a \leqslant x \leqslant b) \\ 1 & (b \leqslant x \leqslant c) \\ \frac{d-x}{d-c} & (c \leqslant x \leqslant d) \end{cases} \quad (1)$$

where $[b,c]$ is called a mode interval of $\tilde{m}$, a, and b are called lower and upper limits of $\tilde{m}$, respectively (Soheil and Kaveh, 2010).

Let $\widetilde{A}$ and $\widetilde{B}$ be two positive trapezoidal fuzzy numbers parameterized by (a_1, a_2, a_3, a_4) and (b_1, b_2, b_3, b_4), then the operational laws of these two trapezoidal fuzzy numbers are follows (Soheil and Kaveh, 2010):

$$\widetilde{A} \oplus \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \quad (2)$$

$$\widetilde{A} - \widetilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4) \quad (3)$$

$$\widetilde{A} \otimes \widetilde{B} = (a_1 \times b_1, a_2 \times b_2, a_3 \otimes b_3, a_4 \times b_4) \quad (4)$$

$$(\widetilde{A})^{-1} = \left(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right) \quad (5)$$