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A study on iterative learning control for impulsive differential equations ☆

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ABSTRACT

In order to tracking the desired discontinuous output trajectory, we explore *P*-type iterative learning control law with initial state learning for impulsive differential equations. The sufficient conditions of open-loop and closed-loop iterative learning schemes in the sense of L^2-norm are established. Finally, an example is given to illustrate our theoretical results.

1. Introduction

It is well known that iterative learning control (ILC for short) is originally explored for robotic applications. This is a methodology applicable to systems which repeatedly perform the same target over a finite time interval. Iterative learning control has been widely investigated in the filed of both theory and applications. There are many significant research results for integer order differential equations, one can refer to the monographs [1–13].

A wide variety of applications can be arisen naturally to explain impulsive differential equations and fractional differential equations such as aircraft control, inspection process in operational research, drug administration and threshold theory in biology. On the one hand, there are many interesting and significants theory results on impulsive differential equations and fractional differential equations, one can see for instance the monographs [14–38]. On the other hand, impulsive control idea is introduced to change the states of a system wherever some conditions are satisfied. In fact, the idea of impulsive control has been demonstrated to be an effective control method through numerical simulations.

Note that the reference trajectory maybe discontinuous functions in many evolution process, iterative learning control methods for precious differential equations can not solve such problem completely. However, there are a few research on iterative learning control for impulsive differential equations. So we pay attention to consider iterative learning control for the following nonlinear impulsive differential equations:

$$\begin{cases} x_k'(t) = f(x_k(t), u_k(t), t), & t \in [0, T] \setminus \{t_1, \ldots, t_m\}, \\ x_k(t_j^+) - x_k(t_j^-) = I_j(x_k(t_j^-)), & j = 1, 2, \ldots, m, \\ y_k(t) = g(x_k(t), u_k(t), t), & \text{a.e. } t \in [0, T], \end{cases} \tag{1}$$

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where $0 < t_1 < \ldots < t_m < T,\ T$ is a pre-fixed number, and $f : \mathbb{R} \times \mathbb{R} \times [0,T] \to \mathbb{R}$, and $g : \mathbb{R} \times \mathbb{R} \times [0,T] \to \mathbb{R}$. Impulsive operators $I_j : \mathbb{R} \to \mathbb{R}$ and fixed time t_j satisfying $0 = t_0 < t_1 < \ldots < t_m < t_{m+1} = T$, the symbols $x(t_j^+) := \lim_{\epsilon\to 0^+} x(t_j + \epsilon)$ and $x(t_j^-) := \lim_{\epsilon\to 0^-} x(t_j + \epsilon)$ represent the right and left limits of $x(t)$ at $t = t_j$ respectively. In general, $x_k(\cdot) \in \mathbb{R}$ is the state of the plant, and $u_k(\cdot) \in \mathbb{R}$ and $y_k(\cdot) \in \mathbb{R}$ denote control input and output, respectively.

The main objective of this paper is to design a P-type ILC law to generate the control input $u_k(\cdot)$ such that the system piecewise continuous output $y_k(\cdot)$ tracks the desired discontinuous output trajectory $y_d(\cdot)$ as accurately as possible as $k \to \infty$ for a.e. $t \in [0,T]$ in the sense of L^2-norm.

The rest of this paper is organized as follows. In Section 2, we give some notations, concepts and preparation results. In Section 3, we give two main results, open-loop and closed-loop iterative learning schemes for impulsive differential equations. An example is given in Section 4 to demonstrate the application of our main results.

2. Preliminaries

Let $J = [0,T]$ and $C(J,\mathbb{R})$ be the space of all continuous functions from J into $\mathbb{R}$ endowed with $\|x\|_C := \sup_{t\in J}\{|x(t)|\}$. We introduce the piecewise continuous functions space $PC(J,\mathbb{R}) := \{x : J \to \mathbb{R} | x \in C((t_k, t_{k+1}],\mathbb{R}),\ k = 0,1,\ldots,m$ and there exist $x(t_k^-)$ and $x(t_k^+),\ k = 1,\ldots,m,$ with $x(t_k^-) = x(t_k)\}$ endowed with $\|x\|_{PC} := \sup_{t\in J}\{|x(t)|\}$. Moreover, we introduce square integrable functions space $L^2(J,\mathbb{R}) := \{y : J \to \mathbb{R} | \int_0^T |y(s)|^2 ds < \infty\}$ endowed with the L^2-norm $\|y\|_{L^2} := \left(\int_0^T |y(s)|^2 ds\right)^{\frac{1}{2}}$. Obviously, $(C(J,\mathbb{R}), \|\cdot\|_C)$, $(PC(J,\mathbb{R}), \|\cdot\|_{PC})$ and $(L^2(J,\mathbb{R}), \|\cdot\|_{L^2})$ are Banach spaces.

Consider the following impulsive Cauchy problems:

$$\begin{cases} x'(t) = f(t,x(t)), & t \in [0,T] \setminus \{t_1,\ldots,t_m\}, \\ x(t_j^+) - x(t_j^-) = I_j(x(t_j^-)), & j = 1,2,\ldots,m, \\ x(0) = x_0. \end{cases} \tag{2}$$

It is well known that if f and I_j satisfy uniformly Lipschitz conditions respectively then one can use standard methods to derive that the problem (2) has a unique solution $x \in PC(J,\mathbb{R})$ which is given by integral equation

$$x(t) = x_0 + \int_0^t f(s,x(s))ds + \sum_{0<t_j<t} I_j(x(t_j^-)), t \in [0,T].$$

To end this section, we collect the following integral inequality of Gronwall type for piecewise continuous functions.

Lemma 2.1 [39]. *Let for $t \geqslant 0$ the following inequality hold*

$$x(t) \leqslant a(t) + b\int_0^t x(s)ds + \sum_{0<t_k<t} \zeta_k x(t_k), \tag{3}$$

where $x,\ a \in PC([0,\infty),\mathbb{R}^+)$, and a is nondecreasing and $b,\ \zeta_k > 0$. Then, for $t \geqslant 0$, the following inequality is valid:

$$x(t) \leqslant a(t) \prod_{0<t_k<t} (1 + \zeta_k)e^{bt}.$$

3. *P*-type ILC for discontinuous trajectory

We introduce the following assumptions:

(H_1) $f : \mathbb{R} \times \mathbb{R} \times [0,T] \to \mathbb{R}$ is strongly measurable for t and are continuous for the first and second variables and there exists a $L_f > 0$ such that

$$|f(x_k(t),u_k(t),t) - f(\bar{x}_k(t),\bar{u}_k(t),t)| \leqslant L_f(|u_k(t) - \bar{u}_k(t)| + |x_k(t) - \bar{x}_k(t)|)$$

for all $x_k,\ u_k,\ \bar{x}_k,\ \bar{u}_k \in \mathbb{R}$ and $t \in J$.

(H_2) $I_k : \mathbb{R} \to \mathbb{R}$ and there exist constants $\rho_k > 0$ such that

$$|I_k(u) - I_k(v)| \leqslant \rho_k|u - v|$$

for all $u,\ v \in \mathbb{R}$ and $k = 1,2,\ldots,m$.

(H_3) For $\beta_j > 0,\ j = 1,2,3,4,$

$$\begin{cases} \beta_1 \leqslant g_u := \frac{\partial g(x_k(t),u_k(t),t)}{\partial u_k(t)} \leqslant \beta_2, \\ \beta_3 \leqslant g_x := \frac{\partial g(x_k(t),u_k(t),t)}{\partial x_k(t)} \leqslant \beta_4, \end{cases}$$