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Fractional order nonlinear systems with delay in iterative learning control ☆

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ABSTRACT

In this paper, we discuss a P-type iterative learning control (ILC) scheme for a class of fractional-order nonlinear systems with delay. By introducing the λ-norm and using Gronwall inequality, the sufficient condition for the robust convergence of the tracking errors is obtained. Based on this convergence, the P-type ILC updating laws can be determined.

1. Introduction

Iterative learning control is suitable for repetitive movements of the controlled system. Its goal is to achieve full range of tracking tasks on finite interval. Iterative learning control (ILC) is one of the most active fields in control theories. ILC belongs to the intelligent control methodology, is an approach for improving the transient performance of systems that operate repetitively over a fixed time interval.

The combination of ILC and fractional calculus was first propose in 2001. In the following ten years, many fractional-order ILC problems were presented aiming at enhancing the performance of ILC scheme for linear or nonlinear systems[2–6]. In recent years,the application of ILC to the fractional-order system has become a new topic. The objective of ILC is to determine a control input iteratively, resulting in plant's ability to track the given reference signal or the output trajectory over a fixed time interval. Owing to its simplicity and effectiveness, ILC has been found to be a good alternative in many areas and applications, see recent surveys for detailed results[7–13].

This paper is on the basis of [14]. We discuss the tracking problem through the open-loop P-type iterative learning control and the closed-loop P-type iterative learning control. The sufficient condition errors with respect to initial positioning error under P-type ILC is obtained by introducing the λ-norm and using Gronwall inequality. The delay part of this paper, we reference[15–20].

In Section 2 some basic definitions of fractional calculus used in this paper are mentioned. Section 3 the main results are shown. Finally, some conclusions are drawn in Section 4.

Throughout this paper, the 2-norm for the n-dimensional vector $w = (w_1, w_2, \ldots, w_n)$ is defined as $\|w\| = (\sum_{r=0}^{n} w_i^2)^{\frac{1}{2}}$, while the λ-norm for a function is defined as$\|\cdot\|_\lambda = \sup_{t\in[0,T]}\{e^{-\lambda t}|\cdot|\}$, where $\lambda > 0$.

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2. Preliminaries

In this section, some basic definitions and lemmas are introduced, which will be used in the following discussions.

Definition 2.1. Riemann–Liouville's fractional integral of order $\alpha > 0$ for a function f : R⁺→ R is defined as

$$ {}_{t_0}D_t^{-\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}f(s)ds. \tag{2.1}$$

Definition 2.2. The Caputo derivatives is defined as

$$ {}_{t_0}^{C}D_t^{\alpha}f(t) = {}_{t_0}D_t^{\alpha-m}D^m f(t), \alpha \in [m-1, m), \tag{2.2}$$

where $m \in Z^+, D^m$ is the classical m-order integral derivative.

Definition 2.3. The definition of the two-parameter function of the Mittag–Leffler type is described by

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}, \quad \alpha > 0, \quad \beta > 0, \quad z \in C. \tag{2.3}$$

For $\beta = 1$ we obtain the Mittag–Leffler function of one-parameter:

$$E_{\alpha}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)}, \quad \alpha > 0, \quad z \in C. \tag{2.4}$$

Lemma 2.4. *The fractional-order differentiation of the Mittag–Leffler function is*

$$ {}_{t_0}D_t^{\gamma}[t^{\beta-1}E_{\alpha,\beta}(\lambda t^{\alpha})] = t^{\beta-\gamma-1}E_{\alpha,\beta-\gamma}(\lambda t^{\alpha}), \quad \gamma < \beta. \tag{2.5}$$

Lemma 2.5. *If the function $f(t,x)$ is continuous, then the initial value problem*

$$\begin{cases} {}_{t_0}^{C}D_t^{\alpha}x(t) = f(t, x_t), \quad 0 < \alpha < 1, \\ x(t_0) = \varphi. \end{cases} \tag{2.6}$$

is equivalent to the following nonlinear Volterra integral equation

$$x(t) = x(t_0) + \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}f(s, x_s)ds. \tag{2.7}$$

and its solutions are continuous [12].

Lemma 2.6. *([14] Generalized Gronwall Inequality). Let $u(t)$ be a continuous function on $t \in [0, T]$ and let $v(t-s)$ be continuous and nonnegative on the triangle $0 \leqslant s \leqslant T$. Moreover, let $w(t)$ be a positive continuous and non-decreasing function on $t \in [0, T]$. If*

$$u(t) \leqslant w(t) + \int_0^t v(t-s)u(s)ds, \quad t \in [0, T], \tag{2.8}$$

then

$$u(t) \leqslant w(t)e^{\int_0^t v(t-s)ds}, \quad t \in [0, T]. \tag{2.9}$$

3. P-type iterative learning control

Consider the following SISO fractional-order nonlinear system with delay

$$\begin{cases} D_t^{\alpha}x^k(t) = f(x_t^k, u^k, t), \\ y^k(t) = g(x_t^k, u^k, t), \end{cases} \tag{3.1}$$

where k is the number of iterations, $k \in \{0, 1, 2, \ldots\}, t \in [0, T], \alpha \in (0, 1)$,