Estimation-based norm-optimal iterative learning control

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\textbf{A B S T R A C T}

The norm-optimal iterative learning control (ILC) algorithm for linear systems is extended to an estimation-based norm-optimal ILC algorithm where the controlled variables are not directly available as measurements. A separation lemma is presented, stating that if a stationary Kalman filter is used for linear time-invariant systems then the ILC design is independent of the dynamics in the Kalman filter. Furthermore, the objective function in the optimisation problem is modified to incorporate the full probability density function of the error. Utilising the Kullback–Leibler divergence leads to an automatic and intuitive way of tuning the ILC algorithm. Finally, the concept is extended to non-linear state space models using linearisation techniques, where it is assumed that the full state vector is estimated and used in the ILC algorithm. Stability and convergence properties for the proposed scheme are also derived.

\section{1. Introduction}

The iterative learning control (ILC) method \cite{1,2} improves performance, for instance trajectory tracking accuracy, for systems that repeat the same task several times. ILC for non-linear systems has been considered in e.g. Avrachenkov \cite{3}; Lin et al. \cite{4}; Xiong and Zhang \cite{5}, where the ILC algorithm is formulated as the solution to a non-linear system of equations. Traditionally, a successful ILC control law is based on direct measurements of the control quantity. However, when the control quantity is not directly available as a measurement, the controller must estimate the control quantity from other measurements, or rely on measurements that indirectly relate to this quantity.

ILC in combination with estimation of the control quantity, has not been given much attention in the literature. In Wallén et al. \cite{6} it is shown that the performance of an industrial robot is significantly increased when an estimate of the control quantity is used instead of measurements of a related quantity. Performance of the ILC algorithm when combined with an estimator has previously been addressed in Axelsson et al. \cite{7}. A related topic has been covered in Ahn et al. \cite{8}; Lee and Lee \cite{9}, where a state space model in the iteration domain is formulated for the error signal, and a $\mathbf{k}_\text{f}$ is used for estimation. The difference to this paper is that in Ahn et al. \cite{8}; Lee and Lee \cite{9}, it is assumed that the control error is measured directly, hence the $\mathbf{k}_\text{f}$ is merely a low-pass filter, with smoothing properties, for reducing the measurement noise.

Here, the estimation-based ILC framework, where the control quantity is not directly available as a measurement, is combined with an ILC design based on an optimisation approach, referred to as norm-optimal ILC \cite{10}. The estimation problem is formulated using recursive Bayesian methods. Extensions to non-linear systems, utilising linearisation techniques, are also presented. The contributions are summarised as

\begin{enumerate}
\item A separation lemma, stating that the extra dynamics introduced by the stationary $\mathbf{k}_\text{f}$ is not necessary to include in the design of the ILC algorithm.
\item Extension of the objective function to include the full probability density function (pdf) of the estimated control quantity, utilising the Kullback–Leibler divergence. This provides an automatic and intuitive choice for one of the weights in the norm-optimal ILC algorithm.
\item Extensions to non-linear systems, including stability and convergence properties.
\end{enumerate}

\section{2. Iterative Learning Control (ILC)}

The ILC-method improves the performance of systems that repeat the same task multiple times. The systems can be open loop as well as closed loop, with internal feedback. The ILC control signal
\( \mathbf{u}_{k+1}(t) \in \mathbb{R}^{n_u} \) for the next iteration \( k+1 \) at discrete time \( t \) is calculated using the error signal and the ilc control signal, both from the current iteration \( k \). Different types of update algorithms can be found in e.g. Moore [2].

One design method is the norm-optimal ilc algorithm [10,11]. The method solves

\[
\min_{\mathbf{u}_{k+1}(t)} \sum_{t=0}^{N-1} \| \mathbf{e}_{k+1}(t) \|^2_{\mathbf{W}_e} + \| \mathbf{u}_{k+1}(t) \|^2_{\mathbf{W}_u} \tag{1}
\]

subject to \( \sum_{t=0}^{N-1} \| \mathbf{u}_{k+1}(t) - \mathbf{u}(t) \|^2 \leq \delta \),

where \( \mathbf{e}_{k+1}(t) = \mathbf{r}(t) - \mathbf{z}_{k+1}(t) \) is the error signal, \( \mathbf{r}(t) \) the reference signal, and \( \mathbf{z}_{k+1}(t) \) the controlled quantity. The matrices \( \mathbf{W}_e \in \mathbb{S}_{++}^{n_e} \), and \( \mathbf{W}_u \in \mathbb{S}_{++}^{n_u} \) are weight matrices, used as design parameters, for the error and the ilc control signal, respectively.

Using a Lagrange multiplier and a batch formulation (see Appendix A) of the system from \( \mathbf{u}_{k+1}(t) \) and \( \mathbf{r}(t) \) to \( \mathbf{z}_{k+1}(t) \) gives the solution

\[
\hat{\mathbf{u}}_{k+1} = \mathbf{Q} \cdot (\hat{\mathbf{u}}_{k} + \mathbf{L} \cdot \hat{\mathbf{e}}_{k}) \tag{2a}
\]

\[
\mathbf{Q} = (\mathbf{T}_{zu}^T \mathbf{W}_e \mathbf{T}_{zu} + \mathbf{W}_u + \lambda \mathbf{I})^{-1}(\lambda \mathbf{I} + \mathbf{T}_{zu}^T \mathbf{W}_e \mathbf{T}_{zu}) \tag{2b}
\]

\[
\mathbf{L} = (\lambda \mathbf{I} + \mathbf{T}_{zu}^T \mathbf{W}_e \mathbf{T}_{zu})^{-1} \mathbf{T}_{zu}^T \mathbf{W}_e \tag{2c}
\]

where \( \lambda \) is a design parameter and

\[
\mathbf{W}_e = \mathbf{I}_n \otimes \mathbf{W}_{e} \in \mathbb{S}_{++}^{n_e}, \quad \mathbf{W}_u = \mathbf{I}_n \otimes \mathbf{W}_u \in \mathbb{S}_{++}^{n_u}. \tag{3}
\]

Under the assumption that \( \mathbf{r}(t) \) does not vary with time, the update Equation (2a) is stable and monotonous for all system descriptions \( \mathbf{T}_{zu} \), i.e., the batch signal \( \hat{\mathbf{u}} \) converges to a constant value monotonically, see e.g. Barton et al. [12]; Gunnarsson and Norrlöf [11] for details of the derivation.

Under the assumption that there are no model uncertainties or noise present, the update Eq. (2a) is stable and monotonous for all system descriptions \( \mathbf{T}_{zu} \), i.e., the batch signal \( \hat{\mathbf{u}} \) converges to a constant value monotonically, see e.g. Barton et al. [12]; Gunnarsson and Norrlöf [11] for details of the derivation.

3. Estimation-based ILC for linear systems

The error \( \mathbf{e}_{k}(t) \) used in the ilc algorithm should be the difference between the reference \( \mathbf{r}(t) \) and the controlled variable \( \mathbf{z}_{k}(t) \) at iteration \( k \). In general the controlled variable \( \mathbf{z}_{k}(t) \) is not directly measurable, therefore an estimation-based ilc algorithm must be used, i.e., the ilc algorithm has to rely on estimates based on measurements of related quantities. The situation is schematically described in Fig. 1.

3.1. Estimation-based norm-optimal ILC

A straightforward extension to the standard norm-optimal ilc method is to use the error \( \hat{\mathbf{e}}_{k}(t) = \mathbf{r}(t) - \hat{\mathbf{z}}_{k}(t) \) in the equations from Section 2, where \( \hat{\mathbf{z}}_{k}(t) \) is an estimate of the controlled variable. The estimate \( \hat{\mathbf{z}}_{k}(t) \) can be obtained using e.g., a Kalman filter (kr) for the linear case, or an extended Kalman filter (ekr) for the non-linear case [13]. Linear systems are covered in this section while Section 4 extends the ideas to non-linear systems. In both cases it must be assumed that (i) the system is observable, and (ii) the filter, used for estimation, converges.

The solution to the optimisation problem can be obtained in a similar way as for the standard norm-optimal ilc problem in Section 2, where the detailed derivation is presented in Amann et al. [10]; Gunnarsson and Norrlöf [11]. An important distinction, compared to standard norm-optimal ilc, relates to what models are used in the design. In the estimation-based approach, the

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1. \( \mathbb{S}_{++} \) denotes a \( n \times n \) positive definite matrix.

2. The stationary Kalman filter uses a constant gain \( \mathbf{K} \) which is the solution to an algebraic Riccati equation, see Kailath et al. [13].
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