



# Estimation-based norm-optimal iterative learning control



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## ABSTRACT

The norm-optimal iterative learning control (ILC) algorithm for linear systems is extended to an estimation-based norm-optimal ILC algorithm where the controlled variables are not directly available as measurements. A separation lemma is presented, stating that if a stationary Kalman filter is used for linear time-invariant systems then the ILC design is independent of the dynamics in the Kalman filter. Furthermore, the objective function in the optimisation problem is modified to incorporate the full probability density function of the error. Utilising the Kullback–Leibler divergence leads to an automatic and intuitive way of tuning the ILC algorithm. Finally, the concept is extended to non-linear state space models using linearisation techniques, where it is assumed that the full state vector is estimated and used in the ILC algorithm. Stability and convergence properties for the proposed scheme are also derived.

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## 1. Introduction

The *iterative learning control* (ILC) method [1,2] improves performance, for instance trajectory tracking accuracy, for systems that repeat the same task several times. ILC for non-linear systems has been considered in e.g. Avrachenkov [3]; Lin et al. [4]; Xiong and Zhang [5], where the ILC algorithm is formulated as the solution to a non-linear system of equations. Traditionally, a successful ILC control law is based on direct measurements of the control quantity. However, when the control quantity is not directly available as a measurement, the controller must estimate the control quantity from other measurements, or rely on measurements that indirectly relate to this quantity.

ILC in combination with estimation of the control quantity, has not been given much attention in the literature. In Wallén et al. [6] it is shown that the performance of an industrial robot is significantly increased when an estimate of the control quantity is used instead of measurements of a related quantity. Performance of the ILC algorithm when combined with an estimator has previously been addressed in Axelsson et al. [7]. A related topic has been covered in Ahn et al. [8]; Lee and Lee [9], where a state space model in the iteration domain is formulated for the error signal, and a

KF is used for estimation. The difference to this paper is that in Ahn et al. [8]; Lee and Lee [9], it is assumed that the control error is measured directly, hence the KF is merely a low-pass filter, with smoothing properties, for reducing the measurement noise.

Here, the estimation-based ILC framework, where the control quantity is not directly available as a measurement, is combined with an ILC design based on an optimisation approach, referred to as norm-optimal ILC [10]. The estimation problem is formulated using recursive Bayesian methods. Extensions to non-linear systems, utilising linearisation techniques, are also presented. The contributions are summarised as

1. A separation lemma, stating that the extra dynamics introduced by the stationary KF is not necessary to include in the design of the ILC algorithm.
2. Extension of the objective function to include the full *probability density function* (PDF) of the estimated control quantity, utilising the Kullback–Leibler divergence. This provides an automatic and intuitive choice for one of the weights in the norm-optimal ILC algorithm.
3. Extensions to non-linear systems, including stability and convergence properties.

## 2. Iterative Learning Control (ILC)

The ILC-method improves the performance of systems that repeat the same task multiple times. The systems can be open loop as well as closed loop, with internal feedback. The ILC control signal

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$\mathbf{u}_{k+1}(t) \in \mathbb{R}^{n_u}$  for the next iteration  $k + 1$  at discrete time  $t$  is calculated using the error signal and the ILC control signal, both from the current iteration  $k$ . Different types of update algorithms can be found in e.g. Moore [2].

One design method is the *norm-optimal* ILC algorithm [10,11]. The method solves

$$\begin{aligned} & \underset{\mathbf{u}_{k+1}(\cdot)}{\text{minimise}} \sum_{t=0}^{N-1} \|\mathbf{e}_{k+1}(t)\|_{\mathbf{W}_e}^2 + \|\mathbf{u}_{k+1}(t)\|_{\mathbf{W}_u}^2 \\ & \text{subject to} \sum_{t=0}^{N-1} \|\mathbf{u}_{k+1}(t) - \mathbf{u}_k(t)\|^2 \leq \delta, \end{aligned} \quad (1)$$

where  $\mathbf{e}_{k+1}(t) = \mathbf{r}(t) - \mathbf{z}_{k+1}(t)$  is the error,  $\mathbf{r}(t)$  the reference signal, and  $\mathbf{z}_{k+1}(t)$  the controlled quantity. The matrices  $\mathbf{W}_e \in \mathbb{S}_{++}^{n_z}$ , and  $\mathbf{W}_u \in \mathbb{S}_{++}^{n_u}$  are weight matrices, used as design parameters, for the error and the ILC control signal, respectively.<sup>1</sup>

Using a Lagrange multiplier and a batch formulation (see Appendix A) of the system from  $\mathbf{u}_{k+1}(t)$  and  $\mathbf{r}(t)$  to  $\mathbf{z}_{k+1}(t)$  gives the solution

$$\bar{\mathbf{u}}_{k+1} = \mathcal{Q} \cdot (\bar{\mathbf{u}}_k + \mathcal{L} \cdot \bar{\mathbf{e}}_k) \quad (2a)$$

$$\mathcal{Q} = (\mathbf{T}_{zu}^T \mathbf{W}_e \mathbf{T}_{zu} + \mathbf{W}_u + \lambda \mathbf{I})^{-1} (\lambda \mathbf{I} + \mathbf{T}_{zu}^T \mathbf{W}_e \mathbf{T}_{zu}) \quad (2b)$$

$$\mathcal{L} = (\lambda \mathbf{I} + \mathbf{T}_{zu}^T \mathbf{W}_e \mathbf{T}_{zu})^{-1} \mathbf{T}_{zu}^T \mathbf{W}_e, \quad (2c)$$

where  $\lambda$  is a design parameter and

$$\mathbf{W}_e = \mathbf{I}_N \otimes \mathbf{W}_e \in \mathbb{S}_{++}^{Nn_z}, \quad \mathbf{W}_u = \mathbf{I}_N \otimes \mathbf{W}_u \in \mathbb{S}_{++}^{Nn_u}, \quad (3)$$

$\mathbf{I}_N$  is the  $N \times N$  identity matrix,  $\otimes$  denotes the Kronecker product,  $\mathbf{T}_{zu}$  is the batch model from  $\mathbf{u}$  to  $\mathbf{z}$ , and  $\bar{\mathbf{e}}_k = \bar{\mathbf{r}} - \bar{\mathbf{z}}_k$ . The reader is referred to Amann et al. [10]; Gunnarsson and Norrlöf [11] for details of the derivation.

Under the assumption that there are no model uncertainties or noise present, the update Eq. (2a) is stable and monotone for all system descriptions  $\mathbf{T}_{zu}$ , i.e., the batch signal  $\bar{\mathbf{u}}$  converges to a constant value monotonically, see e.g. Barton et al. [12]; Gunnarsson and Norrlöf [11].

### 3. Estimation-based ILC for linear systems

The error  $\mathbf{e}_k(t)$  used in the ILC algorithm should be the difference between the reference  $\mathbf{r}(t)$  and the controlled variable  $\mathbf{z}_k(t)$  at iteration  $k$ . In general the controlled variable  $\mathbf{z}_k(t)$  is not directly measurable, therefore an estimation-based ILC algorithm must be used, i.e., the ILC algorithm has to rely on estimates based on measurements of related quantities. The situation is schematically described in Fig. 1.

#### 3.1. Estimation-based norm-optimal ILC

A straightforward extension to the standard norm-optimal ILC method is to use the error  $\hat{\mathbf{e}}_k(t) = \mathbf{r}(t) - \hat{\mathbf{z}}_k(t)$  in the equations from Section 2, where  $\hat{\mathbf{z}}_k(t)$  is an estimate of the controlled variable. The estimate  $\hat{\mathbf{z}}_k(t)$  can be obtained using e.g., a Kalman filter (KF) for the linear case, or an extended Kalman filter (EKF) for the non-linear case [13]. Linear systems are covered in this section while Section 4 extends the ideas to non-linear systems. In both cases it must be assumed that (i) the system is observable, and (ii) the filter, used for estimation, converges.

The solution to the optimisation problem can be obtained in a similar way as for the standard norm-optimal ILC problem in Section 2, where the detailed derivation is presented in Amann et al. [10]; Gunnarsson and Norrlöf [11]. An important distinction, compared to standard norm-optimal ILC, relates to what models are used in the design. In the estimation-based approach, the

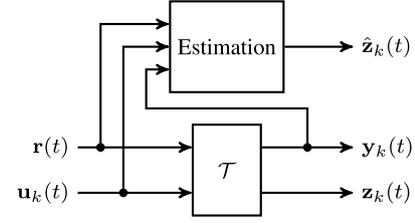


Fig. 1. System  $\mathcal{T}$  with reference  $\mathbf{r}(t)$ , ILC input  $\mathbf{u}_k(t)$ , measured variable  $\mathbf{y}_k(t)$  and controlled variable  $\mathbf{z}_k(t)$  at ILC iteration  $k$  and time  $t$ .

model from  $\mathbf{u}_{k+1}(t)$  and  $\mathbf{r}(t)$  to  $\hat{\mathbf{z}}_{k+1}(t)$  is used, i.e., the dynamics from the KF must be included, while in the standard norm-optimal design, the model from  $\mathbf{u}_{k+1}(t)$  and  $\mathbf{r}(t)$  to  $\mathbf{z}_{k+1}(t)$  is used.

Let the discrete-time state space model be given by

$$\mathbf{x}_k(t+1) = \mathbf{A}(t)\mathbf{x}_k(t) + \mathbf{B}_u(t)\mathbf{u}_k(t) + \mathbf{B}_r(t)\mathbf{r}(t) + \mathbf{G}(t)\mathbf{w}_k(t), \quad (4a)$$

$$\mathbf{y}_k(t) = \mathbf{C}_y(t)\mathbf{x}_k(t) + \mathbf{D}_{yu}(t)\mathbf{u}_k(t) + \mathbf{D}_{yr}(t)\mathbf{r}(t) + \mathbf{v}_k(t), \quad (4b)$$

$$\mathbf{z}_k(t) = \mathbf{C}_z(t)\mathbf{x}_k(t) + \mathbf{D}_{zu}(t)\mathbf{u}_k(t) + \mathbf{D}_{zr}(t)\mathbf{r}(t), \quad (4c)$$

where the process noise  $\mathbf{w}(t) \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}(t))$ , and the measurement noise  $\mathbf{v}(t) \sim \mathcal{N}(\mathbf{0}, \mathbf{R}(t))$ . A batch model (see Appendix A for definitions) for the output  $\mathbf{y}_k$  and the estimate  $\hat{\mathbf{z}}_k$  can be written as

$$\bar{\mathbf{y}}_k = \mathbf{C}_y \Phi \mathbf{x}_0 + \mathbf{T}_{yu} \bar{\mathbf{u}}_k + \mathbf{T}_{yr} \bar{\mathbf{r}}, \quad (5a)$$

$$\hat{\bar{\mathbf{z}}}_k = \mathbf{C}_z \tilde{\Phi} \hat{\mathbf{x}}_0 + \mathbf{T}_{zu} \bar{\mathbf{u}}_k + \mathbf{T}_{zr} \bar{\mathbf{r}} + \mathbf{T}_{zy} \bar{\mathbf{y}}_k, \quad (5b)$$

where  $\mathbf{w}(t)$  and  $\mathbf{v}(t)$  are replaced by the corresponding expected values, which are both equal to zero, in the output model (5a). The KF batch formulation has been used in the model of the estimate in (5b). The optimal solution is now given by

$$\bar{\mathbf{u}}_{k+1} = \mathcal{Q} \cdot (\bar{\mathbf{u}}_k + \mathcal{L} \cdot \hat{\bar{\mathbf{e}}}_k) \quad (6a)$$

$$\mathcal{Q} = (\mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u + \mathbf{W}_u + \lambda \mathbf{I})^{-1} (\lambda \mathbf{I} + \mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u) \quad (6b)$$

$$\mathcal{L} = (\lambda \mathbf{I} + \mathbf{T}_u^T \mathbf{W}_e \mathbf{T}_u)^{-1} \mathbf{T}_u^T \mathbf{W}_e, \quad (6c)$$

where  $\mathbf{T}_u = \mathbf{T}_{zu} + \mathbf{T}_{zy} \mathbf{T}_{yu}$  (see (A.6), (A.10) for details), and  $\hat{\bar{\mathbf{e}}}_k = \bar{\mathbf{r}} - \hat{\bar{\mathbf{z}}}_k$ . The expressions for  $\mathcal{Q}$  and  $\mathcal{L}$  in (6) are similar to (2). The only difference is that  $\mathbf{T}_u$  is used instead of  $\mathbf{T}_{zu}$ . Theorem 1 presents a result for the special case of LTI-systems using the stationary KF.<sup>2</sup>

**Theorem 1** (Separation Lemma for Estimation-Based ILC). *Given an LTI-system and the stationary KF with constant gain matrix  $\mathbf{K}$ , then the matrices  $\mathbf{T}_u$  and  $\mathbf{T}_{zu}$  are equal, hence the ILC algorithm (2) can be used for the estimation-based norm-optimal ILC.*

**Proof.** Assume that  $\mathbf{D}_{yu} = \mathbf{0}$  and  $\mathbf{D}_{zu} = \mathbf{0}$ , then it holds that  $\mathbf{T}_{zu} = \mathbf{C}_z \Psi \mathbf{B}_u$  and  $\mathbf{T}_u = \mathbf{C}_z \tilde{\Psi} \tilde{\mathbf{B}}_u + \mathbf{C}_z \tilde{\Psi}_2 \mathcal{K} \mathbf{C}_y \Psi \mathbf{B}_u$ , see Appendix A. The structure of  $\tilde{\mathbf{B}}_u$  gives

$$\mathbf{T}_u = \mathbf{C}_z (\tilde{\Psi} \Gamma + \tilde{\Psi}_2 \mathcal{K} \mathbf{C}_y \Psi) \mathbf{B}_u,$$

$$\Gamma = \text{diag}(\mathbf{I} - \mathbf{K} \mathbf{C}_y, \dots, \mathbf{I} - \mathbf{K} \mathbf{C}_y, \mathbf{0}).$$

It can now be shown algebraically that  $\tilde{\Psi} \Gamma + \tilde{\Psi}_2 \mathcal{K} \mathbf{C}_y \Psi = \Psi$ , hence  $\mathbf{T}_{zu} = \mathbf{T}_u$ . The case  $\mathbf{D}_{yu} \neq \mathbf{0}$  and  $\mathbf{D}_{zu} \neq \mathbf{0}$  gives similar, but more involved, calculations.  $\square$

The result from Theorem 1 makes it computationally more efficient to compute the matrices  $\mathcal{Q}$  and  $\mathcal{L}$ , since the matrix  $\mathbf{T}_{zu}$  requires fewer calculations to obtain, compared to the matrix  $\mathbf{T}_u$ . Even if the iterative KF update is used, the Kalman gain  $\mathbf{K}$  converges

<sup>1</sup>  $\mathbb{S}_{++}^n$  denotes a  $n \times n$  positive definite matrix.

<sup>2</sup> The stationary Kalman filter uses a constant gain  $\mathbf{K}$  which is the solution to an algebraic Riccati equation, see Kailath et al. [13].

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