Expert Systems with Applications 37 (2010) 3666–3675

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

A neutral DEA model for cross-efficiency evaluation and its extension ☆

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ARTICLE INFO

Keywords:
Data envelopment analysis
Cross-efficiency evaluation
Cross-weight evaluation
DEA ranking

ABSTRACT

Cross-efficiency evaluation has long been suggested as an alternative method for ranking decision making units (DMUs) in data envelopment analysis (DEA). This paper proposes a neutral DEA model for cross-efficiency evaluation. Unlike the aggressive and benevolent formulations in cross-efficiency evaluation, the neutral DEA model determines one set of input and output weights for each DMU from its own point of view without being aggressive or benevolent to the other DMUs. As a result, the cross-efficiencies computed in this way are more neutral, neither aggressive nor benevolent. The neutral DEA model is then extended to a cross-weight evaluation, which seeks a common set of weights for all the DMUs. Numerical examples are provided to illustrate the applications of the neutral DEA model and the cross-weight evaluation in DEA ranking.

1. Introduction

Data envelopment analysis (DEA) developed by Charnes, Cooper, and Rhodes (1978) has been widely accepted as a powerful performance assessment tool, which provides each decision making unit (DMU), a not-for-profit entity or organization that consumes multiple inputs to produce multiple outputs, with a good opportunity to self-evaluate its efficiency relative to the other DMUs. The self-evaluated efficiencies are then compared and ranked. Since the self-evaluation allows each DMU to rate its efficiency with the most favorable weights to itself, more than one DMU is often evaluated as DEA efficient and cannot be discriminated any further. So, lack of discrimination power is one of the major drawbacks that DEA suffers from. It also causes another significant problem that the self-evaluation allows each DMU to be evaluated with its most favorable weights. That is, the inputs and outputs favorable to a particular DMU will be heavily weighted, whereas those not favorable to the DMU will be less weighted or ignored. So, the weights determined by the self-evaluation may sometimes not be realistic.

To increase the discrimination power of DEA and make its weights more realistic, cross-efficiency evaluation has been suggested as an alternative method to the self-evaluation and an extension to DEA. The cross-efficiency evaluation requests each DMU not only to be self-evaluated but also to be peer-evaluated. The concept of cross-efficiency evaluation was first proposed by Sexton, Silkman, and Hogan (1986) and was later examined in detail by Doyle and Green (1994, 1995). In the cross-efficiency evaluation, each DMU determines a set of weights that are either aggressive or benevolent to the others, leading to n sets of weights available for n DMUs. Each DMU is then evaluated with the n sets of weights, respectively, leading to n efficiency values. The n efficiency values for each DMU are finally averaged as an overall efficiency value for the DMU. It is believed that the cross-efficiency evaluation can guarantee a unique ordering for the DMUs and can be used with few DMUs (e.g. four or five) to produce a unique ordering (Doyle & Green, 1995).

Due to its power in discriminating among DMUs, the cross-efficiency evaluation has found a significant number of applications in the DEA literature. For example, Oral, Kettani, and Lang (1991) used the cross-efficiency evaluation for R&D project selection. Shang and Sueyoshi (1995) utilized the cross-efficiency evaluation to select the most efficient flexible manufacturing systems (FMSs). Green, Doyle, and Cook (1996) employed the cross-efficiency evaluation for preference voting and project ranking. Baker and Talluri (1997) applied the cross-efficiency evaluation for industrial robot selection. Talluri and Sarkis (1997) illustrated the use of cross-efficiency for evaluating cellular layouts. Sun (2002) used the cross-efficiency evaluation to differentiate between good and bad computer numerical control (CNC) machines. Chen (2002) used the cross-efficiency evaluation to identify the overall efficient and 'false standard' efficient electricity distribution sectors in Taiwan. Ertay and Ruan (2005) utilized the cross-efficiency evaluation to determine the best labor assignment in cellular manufacturing system (CMS). Lu and Lo (2007a, 2007b) examined the economic-environmental performances of 31 regions in China by taking into

☆ The work described in this paper is supported by the National Natural Science Foundation of China (NSFC) under the Grant No. 70771027.

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doi:10.1016/j.eswa.2009.10.024

account various environmental factors and integrated the cross-efficiency evaluation with cluster analysis to construct a benchmark-learning roadmap for those inefficient regions to improve their efficiencies progressively. Wu, Liang, Wu, and Yang (2008) and Wu, Liang, and Yang (2009a) applied the cross-efficiency evaluation in conjunction with cluster analysis for Olympic ranking and benchmarking.

Apart from the applications mentioned above, theoretical research has also been conducted on the cross-efficiency evaluation. Anderson, Hollingsworth, and Inman (2002) proved in the case of single input and multiple outputs the fixed weighting nature of the cross-efficiency evaluation. They demonstrated with a numerical example how this unseen fixed set of weights might still be unrealistic. Sun and Lu (2005) presented a cross-efficiency profiling (CEP) model which was based upon a combination of the profiling approach developed by Tofallis (1996, 1997) and the cross-efficiency measure presented by Doyle and Green (1994). The CEP model evaluates each input separately and only with respect to the outputs that consume the input. In this way, input-specific ratings based on the cross-efficiency evaluation were derived to give a profile for each DMU. Bao, Chen, and Chang (2008) offered an alternative interpretation to the cross-efficiency evaluation from the viewpoint of slack analysis in DEA. Liang, Wu, Cook, and Zhu (2008b) extended the cross-efficiency model of Doyle and Green (1994) by introducing a number of alternative secondary goals for the cross-efficiency evaluation. Liang, Wu, Cook, and Zhu (2008a) generalized the cross-efficiency concept to game cross-efficiency by viewing each DMU as a player that seeks to maximize its own efficiency under the condition that the cross-efficiency of each of the other DMUs does not deteriorate and the cross-efficiencies as payoffs. A convergent iterative algorithm was presented to derive the best average game cross-efficiency scores, which constitute a Nash equilibrium point. Wu, Liang, and Chen (2009) extended the game cross-efficiency model of Liang et al. (2008a) to variable returns to scale (VRS) and presented a modified DEA game cross-efficiency model by appending an extra constraint to avoid producing negative cross-efficiencies under VRS. The model was then applied to Olympic rankings. Wu (2009) presented a revised benevolent cross-efficiency model and used the cross-efficiencies obtained to construct a fuzzy preference relation instead of the use of average cross-efficiency for ranking DMUs. Wu, Liang, and Yang (2009b) examined the cross-efficiency evaluation from the viewpoint of cooperative game and computed ultimate cross-efficiency by weighting n cross-efficiency values rather than simply averaging them, where the weights for ultimate cross-efficiency were determined by using the Shapley value in cooperative game. Wu, Liang, Yang, and Yan (2009) and Wu, Liang, Zha, and Yang (2009) also developed a bargaining game model and a mixed integer programming model for the cross-efficiency evaluation. In the bargaining game model, each DMU is seen as an independent player and the bargaining solution between the CCR-efficiency and the cross-efficiency are obtained by using the classical Nash bargaining game model. It was shown that the bargaining efficiency was a Pareto solution. The mixed integer programming model was developed to find a best ranking order for each DMU.

From the literature review above, it is found that existing cross-efficiencies except for DEA game cross-efficiency were all computed either aggressively or benevolently. As a matter of fact, there is no guarantee that the two different formulations, aggressive and benevolent, can lead to the same efficiency ranking or decision conclusion. Although most of the applications utilized the aggressive formulation for cross-efficiency evaluation, there is no theoretical evidence to support such a choice. In particular, there have been no attempts so far to test if the two different formulations give the same ranking or conclusion. In this paper, we propose a neutral DEA model for cross-efficiency evaluation to avoid the difficulty in making a choice between the two different formulations. The neutral DEA model determines one set of input and output weights for each DMU without being aggressive or benevolent to the others. As a result, the cross-efficiencies will be more neutral. We then extend the neutral DEA model to determine a common set of weights for all the DMUs, which we refer to as cross-weight evaluation. We also provide a comparison between the cross-efficiency evaluation and the game cross-efficiency evaluation.

The rest of the paper is organized as follows. Section 2 describes the cross-efficiency evaluation and its aggressive and benevolent formulations. The neutral DEA model for cross-efficiency evaluation is developed in Section 3 and extended in Section 4. Comparisons of the cross-efficiency evaluation with the game cross-efficiency evaluation are provided in Section 5. Numerical examples are demonstrated in Section 6. Conclusions are offered in Section 7.

2. The cross-efficiency evaluation

Suppose there are n DMUs to be evaluated against m inputs and s outputs. Denote by x_{ij} $(i=1,\ldots,m)$ and y_{rj} $(r=1,\ldots,s)$ the input and output values of DMU$_j$ $(j=1,\ldots,n)$, whose efficiencies are defined as

$$\theta_j=\frac{\sum_{r=1}^{s}u_r y_{rj}}{\sum_{i=1}^{m}v_i x_{ij}},\quad j=1,\ldots,n, \tag{1}$$

where v_i $(i=1,\ldots,m)$ and u_r $(r=1,\ldots,s)$ are input and output weights.

Consider a DMU, say, DMU$_k$, $k\in\{1,\ldots,n\}$, whose efficiency relative to the other DMUs can be measured by the following CCR model (Charnes et al., 1978):

$$\begin{aligned}\text{Maximize}\quad & \theta_{kk}=\frac{\sum_{r=1}^{s}u_{rk}y_{rk}}{\sum_{i=1}^{m}v_{ik}x_{ik}}\\ \text{Subject to}\quad & \theta_{jk}=\frac{\sum_{r=1}^{s}u_{rk}y_{rj}}{\sum_{i=1}^{m}v_{ik}x_{ij}}\leqslant 1,\quad j=1,\ldots,n,\\ & u_{rk}\geqslant 0,\quad r=1,\ldots,s,\\ & v_{ik}\geqslant 0,\quad i=1,\ldots,m,\end{aligned} \tag{2}$$

which aims to find a set of input and output weights that is most favorable to DMU$_k$.

By using Charnes and Cooper transformation (Charnes & Cooper, 1962), model (2) can be equivalently transformed into the linear program (LP) below for solution:

$$\begin{aligned}\text{Maximize}\quad & \theta_{kk}=\sum_{r=1}^{s}u_{rk}y_{rk}\\ \text{Subject to}\quad & \sum_{i=1}^{m}v_{ik}x_{ik}=1,\\ & \sum_{r=1}^{s}u_{rk}y_{rj}-\sum_{i=1}^{m}v_{ik}x_{ij}\leqslant 0,\quad j=1,\ldots,n,\\ & u_{rk}\geqslant 0,\quad r=1,\ldots,s,\\ & v_{ik}\geqslant 0,\quad i=1,\ldots,m.\end{aligned} \tag{3}$$

Let u_{rk}^* $(r=1,\ldots,s)$ and v_{ik}^* $(i=1,\ldots,m)$ be the optimal solution to the above model. Then, $\theta_{kk}^*=\sum_{r=1}^{s}u_{rk}^*y_{rk}$ is referred to as the CCR-efficiency of DMU$_k$, which is the best relative efficiency that DMU$_k$ can achieve and reflects the self-evaluated efficiency of DMU$_k$. As such, $\theta_{jk}=\sum_{r=1}^{s}u_{rk}^*y_{rj}/\sum_{i=1}^{m}v_{ik}^*x_{ik}$ is referred to as a cross-efficiency value of DMU$_j$ and reflects the peer evaluation of DMU$_k$ to DMU$_j$ $(j=1,\ldots,n;\ j\neq k)$.

Model (3) is solved n times, each time for one different DMU. As a result, there will be n sets of input and output weights available