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Convergence characteristics of PD-type iterative learning control in discrete frequency domain

Xiaoe Ruan*, Zhaozhen Li

School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, PR China

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ABSTRACT

On the basis that a Dirichlet-type signal over a finite time period can be expanded in a Fourier series consisting of fundamental-frequency sinusoidal and cosine waves plus a sequence of higher-frequency harmonic waves, this paper investigates the convergence characteristics of the first- and second-order proportional-derivative-type iterative learning control schemes for repetitive linear time-invariant systems in discrete spectrum. By deriving the properties of the Fourier coefficients in a complex form with respect to the linear time-invariant dynamics and adopting Parseval's Energy Equality, the average energy of the tracking error signal over the finite operation time interval is converted into a quarter of a summation of the fundamental spectrum plus the harmonic spectrums. By means of analyzing the feature of discrete frequency-wise spectrum of the tracking error, sufficient and necessary conditions for monotone convergence with respect to the first-order iterative learning control scheme is deduced together with convergence of the second-order learning scheme is discussed. Numerical simulations manifest the validity and the effectiveness.

1. Introduction

In early 1980s, iterative learning control (ILC) was proposed by S. Arimoto for robot manipulators to precisely track a desired trajectory while it operates repetitively over a fixed time interval [1]. The fundamental mechanism of the ILC is to iteratively generate a control input sequence in a recursive mode by compensating for the current control input with its proportional, integral and/or derivative error so that the input of the next iteration may drive the system to track a desired trajectory as precisely as possible as the number of learning iteration increases. As an intelligent control methodology, it requires less a priori knowledge about the system dynamics. Thus, the ILC techniques and investigations have attracted much attention [2–4], of which the convergence is one of key issues. Particularly, as batch processes control systems are a class of typical repetitive systems, which have widely emerged in the fields such as biochemical and medical industry, electronic manufacturing and polymer materials production and so on, the ILC strategy has been acknowledged as one of powerful techniques for improving the transient performance provided that a conventional PID controller-tuned batch process repetitively attempts to follow an operating trajectory with a desired performance such as quick response, no overshoot or short settling time [5–7].

Regarding to the ILC convergence investigations, the techniques are ranging from time-domain mode to frequency-domain one and others. For the time-domain investigations, in the form of the tracking error evaluated in lambda-norm, it is achieved in papers [8–10] that the convergences of the Proportional-type (P-type), the Derivative-type (D-type) and the Proportional-Derivative-type (PD-type) ILCs are ensured under one of assumptions that the parameter lambda is sufficiently larger, which is irrelevant to the system dynamics, neither the learning gains. As commented in reference [11] that it is the largeness of the non-system parameter lambda that conceals the influence of the system matrix on the convergence, the Arimoto's lambda-norm mode technique for analyzing the convergence needs to be reequipped. By the technique of sup norm, it has investigated in article [12] that the monotone convergence of the PD-type ILC algorithm is only guaranteed in a subinterval. Further, in the form of Lebesgue-*p* norm, reference [13] has conducted that the first-order PD-type ILC is monotonically convergent, whilst the second-order ILC is monotonically convergent after finite iterations, of which the sufficiently convergent condition is not only related to the proportional and derivative learning gains but also the system state matrix as well as the input and output matrices. Up to date, the monotonously convergent assumption is just sufficient.

* Corresponding author. Tel.: +86 013279321898.
E-mail addresses: wruanxe@mail.xjtu.edu.cn (X. Ruan), lzz0427@stu.xjtu.edu.cn (Z. Li).

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In frequency domain, two kinds of techniques are adoptable. One is Fourier series expansion exhibiting that a piece-wise periodic function $f(t)$, $t \in (-\infty, +\infty)$ can be expanded in a Fourier series, which is a summation of fundamental-frequency sinusoidal and cosine waves plus a sequence of harmonic waves with multi-fundamental frequencies. Thus, if a function defined over a finite interval is regarded as an episode of a periodic function, it may be expanded in a Fourier series by the manner. As the Fourier series of a signal characterizes the frequency property of the signal, it is reasonable to utilize the Fourier series as a tool for analyzing the convergence of the ILC rules. In the regard, literature [14] has proposed a fast Fourier transform-type ILC scheme, in which an approximately identified frequency domain model of the system and the inversion of the fast Fourier transform of the error signal are employed for designing the ILC algorithm. The approximation is done by expressing a signal with finite lower-frequency components of the Fourier series. Also, the literature [15] adopted the same approximate technique for expressing the desired trajectory and the output with finite harmonic components of the Fourier series. As such, the derived convergences are basing on the approximation rather than the actual data and thus need to be refined further, though the designed ILC algorithms are feasible in practical executions.

Another frequency domain technique is Laplace transform, which is fit for a dynamical system operating over an infinite time axis. The consequent Laplace transfer function is continuous with respect to the frequency. In specific, the state of a linear time-invariant system can be formulated as a multiplication of the Laplace transfer function and the control input, whilst in time domain it is formulated as a convolution integral of the system state transmit matrix and the control input. As Laplace transform-oriented technique conveys the frequency feature of the system variables, it is preferred for spectrum analysis in practical engineering applications, such as in signal processing and system electric circuit design and so on [16,17]. Fewer of literatures [18,19] have adopted Laplace transform for analyzing the convergence of ILC schemes.

In terms of Laplace transform, the processing is to calculate an abnormal integral of a function $f(t)$ defined over the infinite positive time interval $[0, +\infty)$ expressed as $F(s) = L(f(t)) = \int_0^{+\infty} f(t)\exp(-st)\,dt$ and then to discuss the spectrum function $F(j\omega)$ directly, which is a special case of the Laplace integral $F(s)$ by setting the real part of the complex variable $s = \sigma + j\omega$ to be zero, that is, $Re(s) = \sigma = 0$. This implies that the existence of the Laplace integral $F(s)$ is defaulted for the case when $Re(s) = 0$. But, in a rigorously mathematical derivation, for some functions, the convergence of the abnormal type of Laplace integral $F(s)$ is guaranteed under the assumption that the real part of the complex variable $s = \sigma + j\omega$ is no less than a positive constant, namely, $Re(s) = \sigma \geq c > 0$. Thus, the results in [18,19] for a general case is to some extent controversial to mathematical theory, though the practical application works well. Besides, for a time-varying continuous function over the infinite time interval $[0, +\infty)$ with finite energy, its spectrum is a continuous frequency-varying function. The continuous-frequency Parseval's Energy Theorem says that the energy calculated by integrating the square of the continuous time-varying function over the infinite time interval is equal to integrating the square of the corresponding continuous frequency-varying spectrums over all frequencies [20]. As the concept of Laplace transform for a function over a finite time interval [0,T] is quite different from that over the infinite time interval $[0, +\infty)$, the continuous-frequency Parseval's Energy Theorem for a function over the infinite time interval $[0, +\infty)$ is not appropriate for assessing the ILC tracking performance over a finite time interval. This implies that the existing Laplace transform-based frequency-domain ILC convergence investigations need to be renewed rigorously. The aforementioned queries motivate the paper.

The paper is organized as follows. Section 2 reviews the well-known Dirichlet Theorem concerning that a piece-wise periodic function is decomposed as a Fourier series and then deduces some properties of the Fourier coefficients including discrete frequency-domain Parseval's Energy Equality. Section 3 derives the Fourier series coefficients of the tracking error functions with respect to the first and second-order PD-type ILC schemes. Section 4 analyzes the sufficient and necessary assumptions for the monotone convergence of the proposed ILC algorithms in discrete frequency domain. Numerical simulations manifest the validity and the effectiveness in Section 5 and the last Section 6 concludes the paper.

2. Fourier series and Parseval's Energy Equality

Dirichlet Theorem [21]. If a periodic function $f(t)$, $t \in (-\infty, +\infty)$ with a period T is piecewise monotone on the interval [0,T] and is continuous except possibly for a finite number of discontinuous points of the first type. Then the function $f(t)$ can be decomposed as a Fourier series in a complex form as follows:

$$S(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega t}, \quad (1)$$

where

$\omega = \frac{2\pi}{T}$, $j^2 = -1$, $e^{jn\omega t} = \cos n\omega t + j \sin n\omega t$,

$C_0 = \frac{1}{T}\int_0^T f(t)\,dt$, $C_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega t}\,dt$, $n = \pm1, \pm2, \cdots$,

$$S(t) = \begin{cases} f(t), & \text{if } t \text{ is a point of continuity,} \\ \frac{1}{2}\left[\lim\limits_{\Delta t \to 0^+} f(t+\Delta t) + \lim\limits_{\Delta t \to 0^-} f(t+\Delta t)\right], & \text{if } t \text{ is a point of discontinuity,} \\ \frac{1}{2}\left[\lim\limits_{\Delta t \to 0^+} f(0+\Delta t) + \lim\limits_{\Delta t \to 0^-} f(T+\Delta t)\right], & \text{if } t = 0 \text{ or } T. \end{cases}$$

In the summation (1), the terms $\sin\omega t$ and $\cos\omega t$ produced by $C_{-1}e^{-j\omega t} + C_1 e^{j\omega t}$ are called fundamental-frequency sinusoidal and cosine waves, respectively, whilst $\sin n\omega t$ and $\cos n\omega t$ produced by $C_{-n}e^{-jn\omega t} + C_{+n}e^{jn\omega t}$, for $n = 2, 3, \cdots$, are named as higher-frequency harmonic waves. In usual, the summation $S(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega t}$ is called as a Fourier series expansion of the function $f(t)$ and denoted by $f(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega t}$ for convenience. The above function $f(t)$ is called as a Dirichlet-type function.

Denote the Fourier coefficient C_n of the function $f(t)$ as $F(n\omega)$, that is,

$F(n\omega) = C_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega t}dt$, $n = 0, \pm1, \pm2, \cdots$ Then

$$f(t) = \sum_{n=-\infty}^{+\infty} F(n\omega)e^{jn\omega t}.$$

Thus, $F(n\omega)$, $n = 0, \pm1, \pm2, \cdots$ and $f(t)$ can be regarded as a Fourier pair.

For purpose of simplifying the context statement, in following content all of the equalities and the inequalities with respect to the discrete frequency "$n\omega$" represent that they are fit for all frequency orders $n = 0, \pm1, \pm2, \cdots$.

Lemma 1. (Property 1) If $F_1(n\omega) = \frac{1}{T}\int_0^T f_1(t)e^{-jn\omega t}dt$ and $F_2(n\omega) = \frac{1}{T}\int_0^T f_2(t)e^{-jn\omega t}\,dt$, then

$\alpha F_1(n\omega) + \beta F_2(n\omega) = \frac{1}{T}\int_0^T (\alpha f_1(t) + \beta f_2(t))e^{-jn\omega t}\,dt$, $n = 0, \pm1, \pm2, \cdots$, α and β are constants.

(Property 2) $\frac{1}{T}\int_0^T \frac{df(t)}{dt}e^{-jn\omega t}dt = \frac{1}{T}(f(T) - f(0)) + jn\omega F(n\omega)$.