

# Cooperative Iterative Learning Control of Linear Multi-agent Systems with a Dynamic Leader under Directed Topologies

PENG Zhou-Hua<sup>1</sup> WANG Dan<sup>1</sup> WANG Hao<sup>1</sup> WANG Wei<sup>1</sup>

**Abstract** This paper considers the cooperative tracking of linear multi-agent systems with a dynamic leader whose input information is unavailable to any followers. Cooperative iterative learning controllers, based on the relative state information of neighboring agents, are proposed for tracking the dynamic leader over directed communication topologies. Stability and convergence of the proposed controllers are established using Lyapunov-Krasovskii functionals. Furthermore, this result is extended to the output feedback case where only the output information of each agent can be obtained. A local observer is constructed to estimate the unmeasurable states. Then, cooperative iterative learning controllers, based on the relative observed states of neighboring agents, are devised. For both cases, it is shown that the multi-agent systems whose communication topologies contain a spanning tree can reach synchronization with the dynamic leader, and meanwhile identify the unknown input of the dynamic leader using distributed iterative learning laws. An illustrative example is provided to verify the proposed control schemes.

**Key words** Cooperative control, dynamic leader, linear multi-agent systems, iterative learning

**Citation** Peng Zhou-Hua, Wang Dan, Wang Hao, Wang Wei. Cooperative iterative learning control of linear multi-agent systems with a dynamic leader under directed topologies. *Acta Automatica Sinica*, 2014, **40**(11): 2595–2601

In recent years, compelling attention has been paid to cooperative control of multi-agent systems due to its potential application in numerous engineering areas including unmanned vehicles, manipulators, sensor networks, etc. Along this line of research, consensus as a fundamental control problem is of great importance due to that it is closely related to formation. The key of the consensus is to design a distributed control law based on local interactions with neighbors such that the agents agree on some value of interest<sup>[1–3]</sup>. Numerous results on the consensus control have been obtained; see [4–13] and the references therein. In general, these consensus protocols are falling neatly into two categories, namely, leaderless consensus and leader-follower consensus. In literature, the leader-follower consensus is also called as consensus tracking<sup>[10]</sup>, distributed tracking<sup>[13]</sup>, or cooperative tracking<sup>[12, 14]</sup>.

During the past few years, cooperative tracking of multi-agent systems has been widely studied from different perspectives. In [8], a neighbor-based tracking controller together with a neighbor-based state observer is proposed for each agent to track a dynamic leader. In [9], consensus protocols are developed for first-order linear systems with a time-varying leader dynamics. In [10], robust consensus tracking controllers are proposed for second-order nonlinear systems for both undirected and directed topologies. In [11], consensus of multi-agent systems and synchronization of complex networks are unified in a framework. In [12], a framework for cooperative tracking of linear multi-agent systems is proposed, including state feedback, observer and output feedback. Note that the input is either assumed to be zero<sup>[12]</sup>, or available to a fraction of followers<sup>[9–10]</sup>, or known to all followers<sup>[8, 11]</sup>, which may be restrictive in many circumstances.

To deal with the unknown input of the leader, several methods can be applied<sup>[13–21]</sup>. In [14–18], neural networks are employed to compensate for the unknown input dynamics and unknown individual dynamics. In [19–20], the unknown input of leader can be rejected by a distributed output regulation approach. In [13, 21], the unknown input of leader can be handled using the sliding-mode control approach, which results in a discontinuous controller. Further, the undesirable chattering effect caused by the discontinuous controllers in the previous works<sup>[13, 21]</sup> can be avoided by the proposed continuous controllers in [22].

On the other hand, iterative learning method as an effective control strategy has been widely explored; see [23] and the references therein. The main advantage of iterative learning is using previous information to improve control performance. In [24], iterative learning adaptive law is developed to alleviate the constraint of identifying constant and slowly time-varying parameters. Since the input of the leader may be time-varying and changing very quickly, it is very attractive to apply it to the control of multi-agent systems. However, note that the iterative learning method proposed in [24] is for centralized identification of system parameters in single unknown system, and thus cannot be applied in this case.

Motivated by the above observations, we focus on the cooperative iterative learning control of linear multi-agent systems with a dynamic leader whose input is totally unknown to each agent. The communication topologies among the followers are assumed to be directed and containing a spanning tree. At first, cooperative iterative learning controllers, based on the relative state information of neighboring agents, are proposed. Lyapunov-Krasovskii functional is used to show the uniform ultimate boundedness of closed-loop network signals. Then, this result is extended to the output feedback case, and a local observer is constructed to estimate the unmeasurable states. Based on the relative observed states of neighboring agents, cooperative observer-based iterative learning controllers are devised. A key feature of the proposed controllers allows for tracking the dynamic leader over directed communication graphs without the knowledge of the input.

Manuscript received June 24, 2013; revised September 6, 2013  
Supported by National Natural Science Foundation of China (61273137, 51209026, 61074017), the Scientific Research Fund of Liaoning Provincial Education Department (L2013202), and the Fundamental Research Funds for the Central Universities (3132013037, 3132014047, 3132014321)

Recommended by Associate Editor CHEN Jie  
1. School of Marine Engineering, Dalian Maritime University, Dalian 116026, China

The comparisons with existing works are listed as follows. In contrast to the works<sup>[8–13]</sup>, the input of the leader is not required to be zero, or available to any followers. Compared with the distributed tracking controllers developed in [13, 21], the proposed iterative learning controllers are able to capture the input of the leader, instead of capturing the upper bound of the input. Moreover, the proposed iterative learning control laws are easier to implement in digital processors due to the fact that algebraic equations are used for identification of the dynamics of leader. Unlike the cooperative tracking controllers developed for undirected graphs<sup>[13, 21–22]</sup>, the communication topologies among agents considered here are directed. Finally, it is worth mentioning that the developed protocols are different from the iterative learning controllers proposed in [25–26] in the sense that the control laws are updated in a discrete time form without adaptive terms.

The paper is organized as follows. Section 1 introduces some preliminaries and states the problem formulation. Section 2 presents the state feedback design and stability analysis. Section 3 extends the preceding results to the output feedback case. Section 4 provides an example to illustrate the theoretical results. Section 5 concludes this article.

## 1 Preliminaries and problem formulation

### 1.1 Preliminaries

Consider a network of system consisting of  $N$  follower agents and one leader. If each follower is considered as a node, the neighbor relationship among the followers can be described by a graph  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ , where  $\mathcal{V} = \{n_1, \dots, n_N\}$  is a node set and  $\mathcal{E} = \{(n_i, n_j) \in \mathcal{V} \times \mathcal{V}\}$  is an edge set with the element  $(n_i, n_j)$  that describes the communication from the node  $i$  to the node  $j$ . The neighbor set of the node  $i$  is denoted by  $\mathcal{N}_i = \{j | (n_j, n_i) \in \mathcal{E}\}$ . Define an adjacency matrix  $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N \times N}$  with  $a_{ij} = 1$ , if  $(n_j, n_i) \in \mathcal{E}$  and  $a_{ij} = 0$ , otherwise. Define an in-degree matrix as a diagonal matrix  $\mathcal{D} = \text{diag}\{d_i\} \in \mathbf{R}^{N \times N}$  with  $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$  for node  $i$ . The Laplacian matrix associated with the graph  $\mathcal{G}$  is defined as  $L = \mathcal{D} - \mathcal{A}$ . A directed path in the graph is an ordered sequence of nodes such that any two consecutive nodes in the sequence are an edge of the graph. A digraph has a spanning tree, if there is a node called as the root, such that there is a directed path from the root to every other node in the graph. Finally, define a leader adjacency matrix as  $\mathcal{A}_0 = \text{diag}\{a_{10}, \dots, a_{N0}\}$ , where  $a_{i0} > 0$  if and only if the  $i$ th agent has access to the leader information; otherwise,  $a_{i0} = 0$ . Let  $H = L + \mathcal{A}_0$ .

**Lemma 1**<sup>[14]</sup>. Suppose the graph  $\mathcal{G}$  has a directed spanning tree, and let the root agent has access to the leader. Define

$$\begin{aligned} q &= [q_1, \dots, q_N]^T = H^{-1}\mathbf{1} \\ T &= \text{diag}\{p_i\} = \text{diag}\{1/q_i\}, i = 1, \dots, N \\ G &= TH + H^T T \end{aligned} \quad (1)$$

Then,  $T$  and  $G$  are positive definite.

Throughout the paper, the Euclidean norm and trace are denoted by  $\|\cdot\|$  and  $\text{tr}\{\cdot\}$ , respectively. A diagonal matrix is represented by  $\text{diag}\{b_1, \dots, b_N\}$  with  $b_i$  being the  $i$ th diagonal element. An identity matrix of dimension  $N$  is denoted by  $I_N$ . The Kronecker product is denoted by  $\otimes$ .

### 1.2 Problem formulation

Consider a class of multi-agent systems consisting of  $N$  followers and one leader. The dynamics of the  $i$ th follower is governed by a linear system

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t), i = 1, \dots, N \end{cases} \quad (2)$$

where  $x_i = [x_{i1}(t), \dots, x_{in}(t)]^T \in \mathbf{R}^n$  is the system state;  $u_i(t) \in \mathbf{R}^m$  is the control input;  $y_i(t) \in \mathbf{R}^p$  is the output state;  $A \in \mathbf{R}^{n \times n}$ ,  $B \in \mathbf{R}^{n \times m}$ ,  $C \in \mathbf{R}^{p \times n}$  are known matrices.

The leader is governed by a dynamic system

$$\dot{x}_0(t) = Ax_0(t) + Br(t) \quad (3)$$

where  $x_0 \in \mathbf{R}^n$  is the leader state.  $r(t) \in \mathbf{R}^m$  is an unknown bounded input and may vary quickly in practice. For any initial conditions, assume that the solution  $x_0$  exists for all  $t \geq 0$ .

The control objective of this paper is to design a distributed control law  $u_i$  for each agent (2) to track the leader (3), i.e.,  $x_i(t) \rightarrow x_0(t)$  as  $t \rightarrow \infty$ .

Before designing the controllers, the following assumptions are needed.

**Assumption 1.** The pair  $(A, B)$  is stabilizable.

**Assumption 2.** The pair  $(A, C)$  is detectable.

**Assumption 3.** The time-varying input  $r(t)$  is bounded by  $\|r(t)\| \leq r_M$  with  $r_M$  being a positive constant.

## 2 State feedback

### 2.1 Controller design

In [12], it has been shown that synchronization to the leader (3) with  $r(t) = 0$  can be achieved under directed communication topologies. However, this synchronization controller may not work efficiently if the leader's input is a time-varying trajectory. Motivated by the above observation, a distributed adaptive controller is proposed as follows

$$u_i(t) = cKe_i(t) + \nu_i(t) \quad (4)$$

where  $c \in \mathbf{R}$  is a coupling gain to be specified later;  $K \in \mathbf{R}^{m \times n}$  is a feedback matrix with

$$K = -B^T P \quad (5)$$

where  $P$  is the unique positive definite solution to the following Riccati equation

$$A^T P + PA + Q - PBB^T P = 0 \quad (6)$$

where  $Q \in \mathbf{R}^{n \times n}$  is positive definite;  $e_i(t)$  is a local tracking error defined by

$$e_i(t) = \sum_{j=1}^N a_{ij}[x_i(t) - x_j(t)] + a_{i0}[x_i(t) - x_0(t)] \quad (7)$$

where  $a_{ij}$  are defined in Section 1.1.  $\nu_i(t)$  is used to identify the unknown time-varying input  $r(t)$  that is updated as

$$\nu_i(t) = \kappa_{i1}\nu_i(t - \tau) - \kappa_{i2}p_i(d_i + a_{i0})B^T P e_i(t) \quad (8)$$

where  $\tau \in \mathbf{R}$  is a positive updating interval.  $\kappa_{i1} \in \mathbf{R}^{m \times m}$  and  $\kappa_{i2} \in \mathbf{R}$  satisfy  $0 < \kappa_{i1}^T \kappa_{i1} < \alpha I_m$  with  $0 < \alpha < 1$ , and  $\kappa_{i2} > 0$ , respectively.  $p_i$  is defined in (1).

متن کامل مقاله

دریافت فوری ←

**ISI**Articles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات