Oligopolies with contingent workforce and unemployment insurance systems

Akio Matsumoto [a], Ugo Merlone [b,*], Ferenc Szidarovszky [c]

[a] Department of Economics, Chuo University, 742-1, Higashi–Nakano, Hachioji, Tokyo 192-0393, Japan
[b] Psychology Department, via Verdi 10, University of Torino, Torino I 10124, Italy
[c] Department of Applied Mathematics, University of Pécs, Ifjúság u. 6, Pécs H-7624, Hungary

ARTICLE INFO

ABSTRACT

In the recent literature the introduction of modified cost functions has added reality into the classical oligopoly analysis. Furthermore, the market evolution requires much more flexibility to firms, and in several countries contingent workforce plays an important role in the production choices by the firms. Therefore, an analysis of dynamic adjustment costs is in order to understand oligopoly dynamics. In this paper, dynamic single-product oligopolies without product differentiation are first examined with the additional consideration of production adjustment costs. Linear inverse demand and cost functions are considered and it is assumed that the firms adjust their outputs partially toward best response. The set of the steady states is characterized by a system of linear inequalities and there are usually infinitely many steady states. The asymptotic behavior of the output trajectories is examined by using computer simulation. The numerical results indicate that the resulting dynamics is richer than in the case of the classical Cournot model. This model and results are then compared to oligopolies with unemployment insurance systems when the additional cost is considered if firms do not use their maximum capacities.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

Oligopolies represent one of the most frequently discussed classes of models in the literature of mathematical economics. They have a long history of research starting with the pioneering work of [4]. Oligopoly describes an industry with a moderate number of firms which produce related products or offer related services to a homogeneous market. Most researchers consider this situation as a noncooperative game in which the firms are the players, their production levels or selected prices are the strategies and the profit functions are the payoffs. The existence and uniqueness of the Nash–Cournot or Nash–Bertrand equilibrium was the main research issue in the earlier stages for the classical Cournot model and for its different extensions. This research included single-product models without and with product differentiation, multiproduct oligopolies, labor managed and rent-seeking games to mention only a few. Then the research turned to the dynamic extensions of these models. First linear dynamics were examined, in which local stability implies global stability. The most important results up to the mid 70s are summarized in [14], and their multiproduct generalizations with some applications are presented in [15], where some initial results are also discussed for the nonlinear case. Recently nonlinear models became the major research focus with both discrete and continuous time scales. A duopoly is examined in [7] when one firm uses gradient adjustments and the other firm adaptive adjustments toward best responses and a complete stability analysis is

---

presented based on the speeds of adjustments. Isoelastic inverse demand function is assumed in [8], and two dynamic models are examined: One with simultaneous decisions and the other with sequential decisions. The two dynamic processes are compared and it is shown that the unique equilibrium is asymptotically stable in both cases if the speeds of adjustments are sufficiently small. The duopoly model in [1] assumes cubic cost functions without an inflection point and shows the existence of multiple equilibria with different asymptotic properties. There are several alternative methods to investigate nonlinear dynamics. The Lyapunov approach can be used in both cases, however usually it is very difficult to construct an appropriate Lyapunov function. Linearization can show only local asymptotic stability and provides no or very little information about global asymptotic behavior. In the discrete time scales the critical curve method is a very promising approach, it is practical only for duopolies, symmetric and semisymmetric oligopolies, when the dimension of the corresponding dynamic system is at most two. A comprehensive summary of the newest findings about dynamic nonlinear oligopolies is given in [2].

How much most models discussed in the literature represent real systems is questionable, since they assume homogeneous market, no or fixed capacity limits, no competition of the firms on the secondary market, etc. During the past several years an increasing attention has been given to include additional factors into the classical models.

In this paper we will first include output adjustment costs in addition to the production costs of the firms into their profit functions. Very few studies considered this kind of extension of the classical model. For example, [10] considered the case of linear inverse demand and quadratic production and adjustment costs when the total costs remained differentiable. They showed that the additional cost had no effect on the equilibrium but had a stabilizing effects on the market. In [12] a dynamic game in which the output adjustment cost had only a limited effect on the output levels is introduced. Quadratic adjustment cost functions were considered by [16] without setup cost, so the cost functions remained continuous. The long-term discounted profit was maximized with continuous time scales. In [6] a continuous time scales duopoly in which the output adjustment costs depended on the current output levels and their derivatives is also examined. A discrete time model was analyzed in [19] and similarly to the other studies the cost functions were also continuous. A linear-quadratic differential game model was introduced in [17] with special focus on feedback strategies and equilibria. In [15] both discrete and continuous time scales were considered and the adjustment cost was still continuous at zero, resulting in continuous overall cost functions. Output adjustment and investment costs were combined in [13], in which the best response functions became more complicated but still continuous. The continuity assumption was dropped in [21], best responses of the firms were determined and a complete equilibrium analysis was performed. No dynamic extension of the model was however considered.

In this paper we will first extend the Zhao–Szidarovszky model in a dynamic framework. The payoff functions of the firms are piece-wise differentiable and nonlinear, resulting in nonlinear dynamics. This model will be compared to an alternative approach in which the firms pay some of the unemployment benefit to the unemployed workers. This kind of situation is common in countries such as Italy (see [5]). In the first model output adjustment costs occur when the firms increase output levels, and in the second model the firms face additional expenses when they do not use their total capacity limits. The paper is structured as follows. The mathematical model with output adjustment costs will be first introduced and then the best responses of the firms as well as the Nash equilibria will be determined. The dynamic model will be based on partial adjustment toward best responses with discrete time scales, and the asymptotic properties of this model will be then investigated by computer simulations. And finally, a model with unemployment insurance system will be introduced, analyzed and compared to the nonlinear model of the previous sections. Conclusions and future research directions will be drawn in the last section of the paper.

2. Model with contingent workforce

Consider an industry of N firms that produce identical product or offer identical service to a homogeneous market. Let x_k denote the output of firm k, then the total supply to the market is $X = \sum_{k=1}^{N} x_k$. It is assumed that the inverse demand function of the market is linear: $p(X) = A - BX$ with positive parameters A and B. The cost functions of the firms are also assumed to be linear: $C_k(x_k) = c_k + d_k x_k$ with $d_k > 0$ and $c_k \geqslant 0$. The output adjustment cost at time period t is given as the piece-wise linear function with a positive coefficient γ_k,

$$\bar{C}_k(x_k(t), x_k(t-1)) = \begin{cases} 0 & \text{if} \quad x_k(t) \leqslant x_k(t-1) \\ \gamma_k(x_k(t) - x_k(t-1)) & \text{otherwise}. \end{cases}$$

This kind of piece-wise cost function can describe situations in which the cost of increasing quantities does not include the change of technology or additional investment. This model represents the case of contingent workforce who do not have an implicit or explicit contract for ongoing employment (see, e.g. [20]). Hiring temporary workers can result in increased variable cost, since wages are paid in addition to searching and training costs. Decreasing output level also might generate additional costs. There are different views about the effects of firing costs. For example according to [11], under some conditions flexible wages, can undo the effects of firing costs; for a first discussion see [9]. In Section 5 we will introduce a model in which firms face additional costs by producing less than their capacity limits. However, in the first model we assume the cost arising from decreased output levels are equal to zero.