Journal of Banking & Finance 61 (2015) 327–339

Contents lists available at ScienceDirect

Journal of Banking & Finance

journal homepage: www.elsevier.com/locate/jbf

A utility- and CPT-based comparison of life insurance contracts with guarantees ☆

An Chen, Felix Hentschel, Jakob K. Klein *

Institute of Insurance Science, University of Ulm, Helmholtzstrasse 20, 89081 Ulm, Germany

ARTICLE INFO

Article history:
Received 1 August 2014
Accepted 22 September 2015
Available online 9 October 2015

JEL classification:
G11
G13
G22

Keywords:
Cumulative prospect theory
Life and pension insurance
Portfolio choice
Mortality

ABSTRACT

Some recent literature studies whether contracts including financial guarantees can be preferred by a utility-maximizing investor. The main result for contracts exposed solely to financial risk (e.g. Døskeland and Nordahl, 2008; Dichtl and Drobetz, 2011) is that expected utility theory (EUT) fails to interpret demand for guarantees, while cumulative prospect theory (CPT) is able to support the demand. While the development of the financial market has a significant impact on the portfolio choice, some other non-financial risk might play an important role too. In the present paper, we take life insurance products as an example, where the long term nature of these contracts places a special emphasis on the uncertain lifetime of the investor. We incorporate the mortality risk and investigate its effect on the attractiveness of contracts offering minimum interest rate guarantees to EUT- and CPT-investors. For this purpose, we introduce two approaches for the consideration of multiple risk sources within CPT. We illustrate the theoretical framework with numerous simulations and our numerical results show that a long-term risk-averse EUT-investor prefers products with guarantees.

1. Introduction

The optimal design of life and pension insurance contracts has attracted a lot of attention in the recent literature. In particular, the question whether insurance products with guarantees can be favored is investigated. Related to this question, there are mainly two streams of literature. The first tries to solve an optimization problem for optimal insurance contracts or optimal investment strategies, see e.g. Yaari (1965), Merton (1971), Richard (1975), Raviv (1979), Pliska and Ye (2007), Boyle and Tian (2008), Nielsen and Steffensen (2008), Huang et al. (2008), Bernard and Ghossoub (2010), Bruhn and Steffensen (2011), Sung et al. (2011) and Pirvu and Zhang (2012). The second stream considers several insurance products and tries to interpret the demand for a specific product (among the given products), see e.g. Dierkes et al. (2010), Dichtl and Drobetz (2011), Ebert et al. (2012) and Døskeland and Nordahl (2008). Demand in this context is understood in the sense that an individual will ask for the product that delivers the highest expected utility/value from his point of view.[1] The main common result in the second stream of the literature is: expected utility theory (EUT, mostly taking power utility as an example) is unable to interpret the demand for financial/insurance products which provide a guaranteed payment. However, under cumulative prospect theory (CPT, introduced by Tversky and Kahneman, 1992), the demand for products with guarantees is explainable.

Our paper follows the second stream of literature and in particular the analysis performed by Døskeland and Nordahl (2008). In their setup, they consider four different life/pension insurance contracts, where three contain a guaranteed rate of return and one is the Merton portfolio with respect to terminal wealth (see Merton, 1971). The products with guarantees are: an implicit put with a roll-up guarantee, a simple life contract with a roll-up guarantee and a possible terminal default, as well as a

☆ We would like to thank two anonymous referees whose comments and questions have considerably improved the quality of our paper.

* Corresponding author. Tel.: +49 731 50 31181; fax: +49 731 5031188.
E-mail addresses: an.chen@uni-ulm.de (A. Chen), felix.hentschel@uni-ulm.de (F. Hentschel), jakob.klein@uni-ulm.de (J.K. Klein).

[1] In a purely financial context, Henderson and Pearson (2011) and Benet et al. (2006) approach the problem of explaining an observed demand for structured products by modeling the real products and comparing the performance of fairly priced products with the observed products in terms of expected returns. They stress that an observed demand can be induced by contracts specifications and individual requirements, such as tax advantages and hedging needs, which often remain neglected. In both their and our approach, the phrase "demand" is not the conventional demand function which describes the relationship between quantity and price of a certain good.

http://dx.doi.org/10.1016/j.jbankfin.2015.09.016
0378-4266/

product with an annual guarantee, including the possibility of an annual default. They show that the Merton portfolio outperforms the products with guarantees in terms of expected utility, whereas within CPT, the products with guarantees turn out to be a better choice for the investor. The results of Døskeland and Nordahl (2008) are certainly very interesting. However, in the analysis of demand for insurance contracts with guarantees, they purely consider the financial risks incorporated in these products and neglect another important source of risk: mortality risk. The present paper aims to find out whether by incorporating this risk, the results stated in Døskeland and Nordahl (2008) can be confirmed.

To add mortality risk in the analysis of demand for guarantees, we extend the model of Døskeland and Nordahl (2008) and consider an endowment insurance contract. We then analyze the considered products with EUT and CPT and compare the resulting utility/value levels. An analytical determination of the value function within CPT is impossible due to the options included in the contract design and the possibility of premature payouts.[2] Therefore, we resort to a simulation based analysis to achieve numerical results. Since life/pension insurance contracts are usually long-term contracts, we extend the time horizon to a length that is more realistic for life/pension insurance in our analysis. In addition, we include mortality into CPT with and without the assumption that the individual can assess his survival probabilities realistically. In this sense, we specify the computation of the CPT value for multiple risk sources.

In order to better compare with Døskeland and Nordahl (2008), we first assume that the insurance company follows a constant mix investment strategy. Moreover, we extend their setting and additionally consider the case in which the company follows a constant proportion portfolio insurance (CPPI) strategy. Both of the strategies result in the same qualitative observations.

For both, EUT- and CPT-maximizers, we see that products including (simple) roll-up guarantees are favored over the contract including annual guarantees. This result is consistent with Døskeland and Nordahl (2008). Inconsistent with their results, we find that guarantees are favored by EUT-maximizers when realistic aspects like mortality and long time horizons are taken into consideration. Studying the links between various parameters and the product chosen by the investor, we observe that risk aversion is the most influential parameter. Yet, high risk-aversion alone without mortality and a long time horizon does not make the investor favor a guarantee. Hence, we see that EUT can in fact capture preferences for guarantees when important characteristics are included in the analysis.

By means of sensitivity analyses, we identify the driving factors influencing the CPT-maximizers decisions. In addition, very interestingly, when using the CPT parameters provided in Tversky and Kahneman (1992), instead of the ones chosen by Døskeland and Nordahl (2008), the demand for guarantees cannot be interpreted by CPT. In total, our results raise the question of how general CPT can interpret the demand for guarantees.

The remainder of this paper is organized as follows. In Section 2, we introduce our model setup and particularly specify the payoffs of the contracts. Section 3 introduces the expected utility theory (EUT) and cumulative prospect theory (CPT) on which the liability holder bases his decision. Section 4 provides numerical results and shows the impact of key parameters on the optimal choice. In Section 5, the assumption that the insurance company follows a constant-mix strategy is released. The robustness of our results is examined for constant proportion portfolio insurance strategies. Finally, Section 6 concludes.

2. Model setup

We consider an endowment insurance contract of a single liability holder. We assume the insurance contract is issued at time $t_0 = 0$. At time 0, the liability holder provides an upfront premium payment of L_0 to the insurance company. The company receives an amount of initial contributions E_0 from the equity holder at time 0. Consequently, the initial asset value of the company is given by the sum of the contributions from both the liability holder and the equity holder, i.e. $A_0 = L_0 + E_0$. Furthermore, we denote the upfront premium as a fraction α of the total assets, i.e. $L_0 = \alpha A_0$ with $\alpha \in (0,1)$. The insurance company invests the proceeds in a diversified portfolio of risky and non-risky assets.

The contract has a finite maturity date T, which can be e.g. considered as the retirement date of the liability holder. In case of an endowment life insurance contract, the liability holder receives payments both upon survival of the maturity date T and upon premature death. We use Φ_L to denote the terminal contract payoff of the liability holder:

$$\Phi_L = \mathbb{1}_{\{\tau_x > T\}} \cdot \Psi_L(A_T) + \sum_{t=1}^{T} \mathbb{1}_{\{t-1 < \tau_x \leqslant t\}} \cdot \Psi_L(A_t) \cdot e^{r(T-t)}, \tag{1}$$

where τ_x denotes the remaining life time of the contract holder (with an age x at the contract-issuing time) and $\mathbb{1}_A$ denotes the indicator function which is 1 if A occurs and 0 otherwise. If the contract holder survives the maturity date T, he receives $\Psi_L(A_T)$. If the liability holder dies within $(t-1, t], t = 1, \ldots, T$, the contract payoff $\Psi_L(A_t)$ follows at the end of the period (time t). This form of discretizing the payout, depending on the insured event (in this case mortality), is a common way in insurance, see e.g. Bauer et al. (2008). Usually, insurance contracts are designed in the way that the payout is done at the end of the year in which the insured event (in this case death of the insured) takes place. The reason for this assumption is that one would expect that the event-occurrence-time and the actual payout time are different, because some time is required e.g. by dependents to report the death of the insured, or by the insurance company to actually perform the required transactions, etc. With these considerations, a payout at the end of the year seems a realistic assumption within our framework.[3] To make payments comparable, we assume that all the death payments will be invested in the risk-free asset until maturity, with a constant rate of return r, as in Bauer et al. (2008). Fig. 1 illustrates the time line of the payment upon premature death between $(t-1, t]$. Furthermore, we allow both the premature and maturity payoffs to depend on the evolution of the insurance company's assets. These payoffs will be specified in Section 2.2.

2.1. Underlying financial and demographic risks

Assume a financial market in which there are two traded investment opportunities: a risky and a risk-free asset (bank account). The traded risky asset S satisfies

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where W_t is a standard Brownian motion on our probability space $(\Omega, \mathcal{F}, \mathbb{P})$ ($\mathbb{P}$ is the market probability measure), i.e. this asset follows Black–Scholes dynamics with an instantaneous rate of return $\mu > 0$ and a constant volatility $\sigma > 0$. Also assume the existence of a risk-free asset R which satisfies

$$dR_t = rR_t dt.$$

[2] For a single payout, analytic solutions for portfolio choice problems within CPT have been given in the literature, e.g. Bernard and Ghossoub (2010) and He and Zhou (2011).

[3] For a continuous payout, immediately at the time of death, we would have $\Phi_L = \Psi_L(A_T) \cdot \mathbb{1}_{\{\tau_x > T\}} + \Psi_L(A_{\tau_x}) e^{r(T-\tau_x)} \cdot \mathbb{1}_{\{0 < \tau_x \leqslant T\}}$. The model is then slightly modified, but still yields similar numerical results.