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Discrete Optimization

The relevance of the "alphorn of uncertainty" to the financial management of projects under uncertainty

Jingwen Zhang [a,b,*], Salah E. Elmaghraby [b]

[a] School of Management, Northwestern Polytechnical University, Xi'an 710072, PR China

[b] Department of Industrial and System Engineering, North Carolina State University, NC 27695, USA

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ABSTRACT

In this paper, both the duration and the cost of an activity are modeled as random variables, and accordingly, the cumulative cost at each time point also becomes a random variable along a project's progress. We first present the new concept of an "*alphorn of uncertainty*" (*AoU*) to describe the domain of cumulative cost variation throughout the life of a project and subsequently apply it to assess the project's financial status over time. The shape of the alphorn was obtained by mixing Monte Carlo sampling with Gantt chart analysis, which enabled us to determine a project's financial status related to specific payment modes. To validate the *AoU*, we designed and conducted an extensive numerical experiment using a randomly generated data set of activity networks. The results indicate that the *AoU* may be a promising method for the financial management of projects under uncertainty. Furthermore, financial status under uncertain conditions is not sensitive to an activity's choice of duration distributions or to the form of cost functions. However, payment rules can greatly affect financial status over the duration of a project.

1. Introduction

Projects are often subject to considerable uncertainty during their execution, which explains why activity durations are treated as random variables (*r.v.s*) in project scheduling problems. While much has been published regarding uncertainty in activity durations (Ballestin & Leus, 2009; Klerides & Hadjiconstantinou, 2010; Sobel, Szmerekovsky, & Tilson, 2009; Zhang, Tam, & Li, 2005; Zhu, Bard, & Yu, 2007), little has been conducted regarding other important parameters involved in a project's progress that are greatly impacted by stochastic activity durations—in particular, activity costs, cumulative costs over time and profitability—and it is therefore reasonable to conclude that these parameters are also *r.v.s*. Unfortunately, discussions of cost or gain are usually conducted in a deterministic setting and the objective aims at minimizing total cost or at maximizing net present value (*NPV*). Clearly, total cost or *NPV* is only a performance indicator at the completion of a project, but financial status cannot be monitored while a project is in progress. That notwithstanding, project managers are often concerned with the financial status of pending projects, especially large-scale projects that continue over a long period of time (so-called 'mega-projects', see Flyvbjerg, Bruzellius, & Rothengatter, 2003). At different stages of project execution, contractors' decisions are largely influenced by financial status. In reality, even some high-profit projects that are evaluated in the planning phase may turn out to be financial disasters due to sudden events occurring during the course of their implementations, such as cash shortages, equipment failures, and under- or over-estimation of task contents. Projects in uncertain environments may experience similar conditions more frequently. This embarrassing situation motivated us to investigate the effect of cost progress on financial risks during the execution of a project under uncertainty.

In this paper, we address the assessment of a project's profits or losses over its life cycle under uncertainties involving both activity duration and cost. We submit that although the issue of time schedules has been extensively investigated in project scheduling literature, the cost progress on accompanying such schedules has largely been neglected. To the best of our knowledge, research on this topic has been scant. There are two research streams about project scheduling problems in deterministic settings that relate to this issue: the max-*NPV* and the earned value method (*EV*/*EVM*). Therefore, we briefly review these two streams from the perspective of cost progress.

In max-*NPV* and payment scheduling studies, a series of cash flows occurs as a project progresses. As contractors are induced

* Corresponding author at: School of Management, Northwestern Polytechnical University, Xi'an, 710072, PR China. Tel.: +86 029 88431781, tel.: +86 15229304613.
E-mail address: zhangjingwen@nwpu.edu.cn (J. Zhang).

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List of acronyms

AoU	alphorn of uncertainty	*l.b.*	lower bound
AN	activity network	*u.b.*	upper bound
CNV	cumulative net value	*MCS*	Monte Carlo sampling
EVM	earned value method	*r.v.*	random variable

to make expenditures through the use of various resources while activities are being executed, cost progress is involved, to a slight degree, in max-*NPV*. Because the time value of capital is considered when taking *NPV* as the performance measure, the time at which activity costs are disbursed will influence a project's *NPV*. In the context of the max-*NPV*, activity expenses are differentiated into two types of cases. The first type of case is one in which the total cost is proportionally allocated to an activity's beginning and ending events (Dayanand & Padman, 2001; He, Wang, Jia, & Xu, 2009), which can be easily understood by an activity-on-arc (AoA) representation. Alternatively, the total cost of an activity is assumed to have been incurred at either its beginning (Padman & Zhu, 2006; Szmerekovsky, 2005; Ulusory, Sivrikaya-Serifoglu, & Sahin, 2001) or its completion (Seifi & Tavakkoli-Moghaddam, 2008; Vanhoucke, Demeulemeester, & Herroelen, 2003; Varma, Uzsoy, Pekny, et al., 2007). The second type of case is one in which an activity's total cost is uniformly (or unevenly) distributed as it progresses (Erenguc, Tufekci, & Zappe, 1993; Kavlak, Ulusoy, Sivrikaya-Serifoglu, & Birbil, 2009). Additionally, Chen, Zhang, Chung, et al. (2010) have stated that the constant cost for an activity arises at the beginning and that resource costs are incurred as resources are either occupied or consumed. Herroelen, Van Dommelen, and Demeulemeester (1997) gave an earlier review of the max-*NPV* with discounted cash flow. For recent comments on the max-*NPV*, we refer readers to Drezet (2008), Hartmann and Briskorn (2010), Weglarz, Jozefowska, Mika, et al. (2011), and He, Liu, and Jia (2012). As the max-*NPV* and payment scheduling problems involve payment behaviors between contractors and owners, methods of disbursing activity costs are of little concern.

The issues discussed in this paper are related to the concept of the *EVM*. In the past decade, the *EVM* in deterministic settings has received extensive coverage (Goodpasture, 2004; Marshall, 2007; Solomon & Young, 2006; Philipson & Antvik, 2009), but the uncertain nature of an activity's progress is neglected in the traditional *EVM*. Additionally, probabilistic forecasting of project performances at completion is based on the concept of an S-curve in the *EVM* (Barraza, Edward Back, & Meta, 2004; Kim & Reinschmidt, 2009), while Naeni, Shadrokh, and Salehipour (2011) have discussed the *EVM* based on the fuzzy theory. However, the several researchers cited above have addressed predictions of project performance at completion, which do not involve either stochastic cumulative cost or financial status while a project is in progress. We share the basic tenet of the *EVM*, but note that we differ from the *EVM* in that we focus on at least two important issues. First, we take into account the inherent uncertainty of conducting activities; and second, we introduce multiple methods of expenditure among activities.

The remainder of this paper is organized as follows. Section 2 contains a description of the problem, assumptions and notations. In Section 3, we establish the rationale for the "alphorn of uncertainty" (*AoU*) and argue for the use of Monte Carlo sampling (*MCS*) to secure the *AoU*. This section also presents the application of the *AoU* to financial assessments associated with two payment modes during the execution of a project. Section 4 designs a full numerical experiment to gain new insights for projects' financial statuses related to stochastic activity durations and costs. Conclusions and future extensions are presented in Section 5.

2. Problem formulations

A project's activity network (*AN*) is represented by a directed acyclic graph. *V* represents the set of non-dummy activities in an *AN*, and $|V| = J$. Zero-lag finish-start precedence constraints between activities are present. Some labels and notations are listed below to set up our problem,

j: index of activities in an *AN*, $j = 1, 2, \ldots, J$;
$P(j)$ $\{S(j)\}$: the set of immediate predecessors {successors} of activity j;
$\tilde{d}_j$: the duration of activity j ($j = 1, 2, \ldots, J$), which is assumed to be a *r.v.*, and $\tilde{d}_j \in [a_j, b_j]$, a_j and b_j are the largest and the smallest values of $\tilde{d}_j$, respectively;
$f(\tilde{d}_j)$: the probability distribution for $\tilde{d}_j$;
$\widetilde{C}_j$: the total cost incurred to complete activity j, which depends on its stochastic $\tilde{d}_j$;
$\tilde{s}_j\{\tilde{f}_j\}$: the beginning {ending} time of activity j in the critical path method (CPM), with $\tilde{s}_i + \tilde{d}_i = \tilde{f}_i \leqslant \tilde{s}_j$, $i \in P(j)$;
$\widetilde{T}$: the project duration under stochastic durations of activities is composed of *r.v.*s, and $T_{min} \leqslant \widetilde{T} \leqslant T_{max}$, where T_{max} and T_{min} are the longest and the shortest project durations, respectively.
t: the time variable over the progress of a project, thus $0 \leqslant t \leqslant \widetilde{T}$;
$\widetilde{C}_j(t) = y(\widetilde{C}_j, \tilde{d}_j, t); \tilde{s}_j \leqslant t \leqslant \tilde{f}_j$ and $0 \leqslant \widetilde{C}_j(t) \leqslant \widetilde{C}_j$: the function of cost disbursement for activity j, assuming that $\widetilde{C}_j$ is gradually disbursed as activity j progresses. The expenditure for an activity is incurred only after it is started, and the overall cost has just been expended at its end. $\tilde{c}_j(t)$ is the disbursed cost when activity j has been executed for $(t - \tilde{s}_j)$ time units, namely if $t < \tilde{s}_j$, then $\tilde{c}_j(t) = 0$; if $t \geqslant \tilde{f}_j$, then $\tilde{c}_j(t) = \widetilde{C}_j$.

Naturally, the cumulative cost at time t_f certainly is a *r.v.*, which is expressed by $\widetilde{UC(t_f)}$ in expression (1):

$$\widetilde{UC(t_f)} = \sum_{\substack{i \in V \cap \\ t_f \geqslant \tilde{f}_i}} \widetilde{C}_i + \sum_{\substack{j \in V \cap \\ \tilde{s}_j < t_f < \tilde{f}_j}} \tilde{c}_j(t_f - \tilde{s}_j) \tag{1}$$

$t_f \geqslant \tilde{f}_j$ means that activity i has been completed at $t = t_f$, and $\tilde{s}_j < t_f < \tilde{f}_j$ indicates that activity j is in process at $t = t_f$. $\widetilde{UC(t_f)}$ at each time t_f consists of two parts, one that is the sum of the costs resulting from completed activities at t_f and another that is the sum of the costs incurred by executing activities at t_f.

Under one realized scenario of a project, the cumulative cost curve over time t is generally shown by expression (2):

$$UC(t) = \sum_{i \in V \text{ and } t \geqslant f_i} C_i + \sum_{j \in V \text{ and } s_j < t < f_j} c_j(t - s_j) \tag{2}$$

There are many realized scenarios for cumulative cost curves over the duration of a project due to stochastic durations and costs of activities, which accordingly form a cluster of cumulative cost curves. The upper bound (*u.b.*) and the lower bound (*l.b.*) on the cumulative cost curves at each time t_f during project execution can be described mathematically as follows: