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Forecasting demographic forecasts

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ABSTRACT

Consider the financial sustainability of public finances in the context of stochastic demographics. Such analyses have typically been made under the assumption that future demographic developments are deterministic. When stochastic demographics have been considered, the problems have been simplified by assuming that the decision makers in the economic system behave as if they had perfect foresight as regards demographics. More realistically, we assume that the decision makers base their decisions on the forecasts of the future population, but revise their decisions when it turns out that the demographics do not follow the expected path. We contrast the nature of demographic uncertainty with that of financial markets, and argue that it is not realistic to assume that the revisions will occur according to the full rational expectations paradigm. Instead, the decision makers are assumed to revise according to the most recent point forecast. To implement this approach, we tailor standard nonparametric regression techniques to the task of computing the required future conditional expectations. Specifically, we assume that an approximation to the predictive distribution of the future population is available in terms of simulated population counts. The required conditional expectations are then obtained by averaging the future evolution of a set of sample paths that come from the neighborhood of a target path. This is formally equivalent to n-nearest neighbor kernel regression. The degree of smoothing can be chosen via cross-validation. An illustration based on a stochastic forecast of the population of Finland is given.

1. Introduction

The problem of how to best anticipate demographic forecasts that are only made in the future arises naturally in the analysis of fiscal sustainability. Consider discrete time and let $t = 0$ correspond to the current year, when a forecast of the future population is made. Our decisions concerning current and future labor demand and supply depend on these forecasts, as they tell us what the future potential labor supply at different ages will be. If, by year $T > 0$, the population development has deviated from the forecast, we might like to adjust our decisions. This would involve making a new forecast of the population for $T + 1$, $T + 2, \ldots$. Thus, we wish to anticipate at $t = 0$ what the forecast at $t = T$ or later will be, under any population development during $0 \le t \le T$.

More formally, a forecast made at time t conditions on information available up to that time. Let us denote by $\mathbf{Y}(t, k)$ the vector of demographic variables whose values become known during the years $t, t + 1, \ldots, k$, for $k > t$. Then, under squared error loss,[1] the initial point forecast at $t = 0$ would be the expectation $E[\mathbf{Y}(1, H)]$, given what

[1] If the predictive distribution of future values has fat tails, then the absolute value loss may be more appropriate. In this case, the optimal prediction would be the median. Moreover, asymmetric loss functions may be desirable in specific applications. However, as one person's economic loss is often another person's gain, care is needed in the interpretation of the resulting forecasts.

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was known at $t = 0$ for some lead time of interest $1 \leq H \leq +\infty$. The updated point forecast at a future time $t = T$ would then be the conditional expectation $E[\mathbf{Y}(T+1, H) \mid \mathbf{Y}(0, T)]$. For reasons outlined by Land (1986), demographic forecasts are nearly universally made based on demographics alone, so the fact that the conditioning is with respect to demographic variables only is not a major limitation.

Our topic is closely related to the topic of forecast updating for time series analysis (e.g., Box & Jenkins, 1976, p. 164). However, those discussions typically assume that a well-identified analytical model is available, which is not the case in our applications. From a more formal perspective, our work can be viewed as an instance of the prequential[2] theory of Dawid (1984). The primary contributions of this paper are (1) to discuss the relevance of the updated forecasts $E[\mathbf{Y}(T+1, H) \mid \mathbf{Y}(0, T)]$ in macroeconomic decision making and (2) to show how these can be computed when only a numerical approximation, based on stochastic simulation, is available for the predictive distribution of $\mathbf{Y}(1, H)$. We will adapt standard nonparametric regression methods (e.g., Härdle, 1994) for the latter task.

In Section 2, we will sketch the structure of the macroeconomic models for which the results are needed, and provide a motivation for the use of updated forecasts. Section 3 describes the recursive nature of population renewal. Section 4 discusses the nature of the nonparametric calculations. In particular, the bias and the variance of the nonparametric estimators are illustrated, and the selection of tuning parameters via cross validation is discussed. In Section 5, the methods are illustrated by using data from a recent stochastic population forecast of Finland. More details of the technical aspects, and some R code (cf. http://cran.r-project.org/), are given in the report by Alho (2014). We conclude in Section 6 with a brief discussion of the implications of the proposed approach.

The mathematical formalism is kept to a minimum. Random variables and vectors are given in upper case: Y, $\mathbf{Y}, \ldots$; their values are given in lower case: y, $\mathbf{y}, \ldots$; and time t dependent demographic vectors and matrices are written as $\mathbf{V}(t)$, $\mathbf{R}(t), \ldots$.

2. Economics and changing demographics

2.1. Overlapping generations models

The problem of sustainability involves an infinite horizon: can current policies be continued indefinitely without imposing an undue burden on the participants of the labor market? In so-called *generational accounting* (cf. Auerbach, Kotlikoff, & Leibfritz, 1997), the starting point is that current age and sex-specific tax and entitlement rates are assumed to be fixed forever. A single "best estimate" path of future demographic development is specified, and one determines whether or not current tax rates cover the current entitlements for this path. As was discussed by Alho and Vanne (2006), it is possible to incorporate the uncertainty in the future population and economic development into such calculations via stochastic simulation. However, since tax and entitlement rates are assumed to be exogenous, there is no economic adjustment to alternative future paths.

In contrast, in the so-called *overlapping generations* (OLG) model (Samuelson, 1958), decision makers consider the consumption and saving behaviors of all current and future birth cohorts. In the Finnish Overlapping Generations model of Lassila, Palm, and Valkonen (1997), for which our specific results are tailored, there is a profit-maximizing firm that pays wages to the labor force. In return, the labor force agrees to forgo leisure in order to finance consumption. The price of labor depends on the size of working age cohorts. Similarly, consumption before and after working age is financed either by borrowing from those working or by saving. Thus, the optimal labor supply, wages, and consumption decisions of all currently living and future cohorts are interlinked. In a deterministic setting, the optimal wages, labor expended, and consumption are decided in a single optimization step, under the assumption that the future population will follow the best estimate path $E[\mathbf{Y}(1, +\infty)]$.

Alho, Hougaard-Jensen and Lassila (2008) give several applications of OLG models, in which stochastic population evolution is allowed. This is accomplished by taking repeated samples (typically 3000) from the distribution of $\mathbf{Y}(1, H)$, and leads one to consider a much richer spectrum of alternative future developments than a single point forecast, meaning that any conclusions regarding sustainability rest on a firmer footing. However, new conceptual problems also arise. When the OLG model is solved path by path, this can be interpreted as saying that the decision makers in fact possess the gift of *perfect foresight* as regards demographics.

2.2. Perfect foresight vs. realistic levels of information

Alho and Määttänen (2008) compared the perfect foresight assumption to three alternative cases in which the decision maker knows less, in the context of a life-cycle saving problem. The only source of uncertainty was future mortality.

Suppose that the actual lifetime U has the survival function $p(u) = P(U > u)$. It is not realistic to think that a decision maker would know the actual value of U, but Alho and Määttänen (2008) considered the survival probability itself to be random, and distinguished four cases.

(a) In the case of perfect foresight, the decision maker was assumed to know the exact future path of $p(u)$.
(b) In the full *rational expectations* case (cf. Muth, 1961), she was assumed to know the exact probability distribution of $p(u)$, but not the mortality path that she will be exposed to.
(c) In the intermediate information case she was given an updated forecast of $p(u)$, say, every five years $E[p(5i + H) \mid p(j), j \leq 5i]$ for all $i = 1, 2, \ldots$ and $H > 0$. For lack of a better term, we call this the *updated forecasts* case.
(d) In the lowest information case, the decision maker was given only a point forecast $E[p(u)]$ at $t = 0$.

Perfect foresight leads to the highest level of welfare, when averaged over the demographic outcomes. Rational expectations comes second, updated forecasts third, and

[2] Short for probability forecasting with sequential prediction.