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Micro to macro models for income distribution in the absence and in the presence of tax evasion

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ABSTRACT

We investigate the effect of tax evasion on the income distribution and the inequality index of a society through a kinetic model described by a set of nonlinear ordinary differential equations. The model allows to compute the global outcome of binary and multiple microscopic interactions between individuals. When evasion occurs, both individuals involved in a binary interaction take advantage of it, while the rest of the society is deprived of a part of the planned redistribution. In general, the effect of evasion on the income distribution is to decrease the population of the middle classes and increase that of the poor and rich classes. We study the dependence of the Gini index on several parameters (mainly taxation rates and evasion rates), also in the case when the evasion rate increases proportionally to a taxation rate which is perceived by citizens as unfair. Finally, we evaluate the relative probability of class advancement of individuals due to direct interactions and welfare provisions, and some typical temporal rates of convergence of the income distribution to its equilibrium state.

1. Introduction

The rise of inequalities in income and wealth, the implementation of different tax policies and the effects of tax evasion constitute important socioeconomic questions for most countries. Especially in times of economic crisis, like the current one, such issues become of major concern and are the object of frequent studies and debates. Involving a large number of interacting agents, as well as a multiplicity of aspects and levels, this matter certainly falls within the realm of the science of complex systems. We think that also mathematics can contribute to some extent to the analysis of these problems; for example, it can help to understand the micro-processes and mechanisms which lead to certain collective patterns. Through modelling and simulations, made possible by the power of modern computers, mathematics allows the exploration of several possible scenarios. Thus, in conjunction with the expertise from economics, political economics and other disciplines, and suitably supported by empirical data, mathematical models could in some cases even suggest concrete policies.

Basically motivated by this belief, we consider in this paper some microscopic models of taxation and redistribution in a closed market society, both in the absence and in the presence of tax evasion. These models are constructed within a general framework which was first introduced in [1] and then further investigated in [2,3]. They are formulated as systems of nonlinear ordinary differential equations. More precisely, the systems expressing them consist of a number n of equations equal to the number of income classes in which one divides a population. The jth equation (with $j = 1, \ldots, n$) describes the

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variation in time of the fraction, say x_j, of individuals belonging to the jth class. The vector $(x_1, \ldots, x_n)$ represents the discrete income distribution over the population, whose size is supposed to remain constant. According to the findings of [1–3], in correspondence to any value μ of the total income (which is a conserved quantity too) a stationary income distribution exists, which is the asymptotic trend of all initial distributions with total income μ. In [2,3] it was also shown that, if the number n of classes is large enough (n was taken to be equal to 25 in those papers) and if the value μ is compatible with initial distributions having the majority of individuals in the lower income classes, then for models constructed with suitable parameters (and those discussed here are of this type) the asymptotic income distributions exhibit fat tails with Pareto power-law behaviour like the real world distributions.

The main novelty with respect to the models explored in [1–3] is that we treat here also cases in which tax evasion occurs. This addition is not irrelevant. Indeed, the illegal practice of tax evasion affects probably all societies, causing the "loss" of huge amounts of money, which could be employed towards social and economic policies. We are especially interested in the differences of the asymptotic income distributions in cases of tax compliance and in cases with tax evasion. Below, we investigate these differences and we examine how quantities and indicators like the Gini index, the tax revenue and the probability of class promotion due to welfare change in the various cases. In our approach the aggregate behaviour of a system, represented by the observable income distribution curves, emerges from the complex of interactions which take place between single heterogeneous individuals.

The underlying behaviour- and interaction-based perspective differs intrinsically from the traditional viewpoint of mainstream economics, whose cornerstones are the assumption of a representative agent and the rational choice theory. The interaction-based paradigm began to take shape during the last decades and it counts among its pioneers various exponents of the economics community, e.g. Schelling, Kirman, Arthur, and Gallegati, see e.g. [4–7]. The tool kit of researchers adopting this perspective typically includes agent-based computational simulations and complex networks. For example, questions related to tax evasion have been investigated via agent-based models in [8–10]. In these papers the focus is on the effect of interactions among behaviourally different agent types (honest, imitative, tax evaders and so on) on the changes in individual behaviour patterns. An experimental approach to such kind of questions has been developed and described in [11].

On another side, starting in the mid-1990s a branch of physics denoted econophysics[1] has been developed, which explores the dynamical behaviour of economic and financial markets by means of methods and tools originally developed in statistical mechanics and in gas kinetic theory (see e.g. in this connection [12–19]). In econophysics, taxation and redistribution have been modelled in [13] through a Boltzmann equation and in [20] through a two-steps process involving wealth exchanges similar to inelastic binary "collisions". The effect of subsidies by the government has been found in both papers to lead to a shifting of the individuals in the equilibrium distribution from the lower toward middle income classes. The phenomenon of tax evasion has been described e.g. in [9] through an analogy with the Ising model, which is an array, typically 2-dimensional, of spin variables s_{ij} that interact with their nearest neighbours and can only assume the values ± 1. In the analogy each spin represents a citizen, which can be either in the tax compliant state $+1$ or in the tax evader state -1 and can undergo transitions from $+1$ to -1 due to imitation and from -1 to $+1$ due to tax audits. Through numerical simulations or approximations typical of statistical mechanics it is possible to compute the average $\langle \sum_{ij} s_{ij} \rangle$, directly related to the total evaders/compliant rate, as a function of several global or local parameters. This approach is helpful for the analysis of evasion phenomena in relation to local interaction and external controls, but not for studying the effect of evasion on the income distribution as we do here.

The paper is organized as follows. In the next section we sketch some models, which were constructed and analysed in [1–3], and we recall some of their features as established in these papers. In Section 3 we incorporate into these models the tax evasion phenomenon and we discuss some first results concerning the asymptotic stationary income distributions which are found in the absence and in the presence of tax evasion. We then explore in Section 4 the case in which to an increase of tax rates a proportional increase of evasion corresponds. The fifth and the sixth section are devoted respectively to an in-depth analysis of some interesting quantities characteristic of the stationary solutions and of the relative times of convergence. Some summarizing comments are contained in Section 7.

2. The tax compliance case

We shortly review in this section a family of models regarding a tax compliance case, and we recall their main features as established in [1–3].

Imagine dividing a population of individuals into a finite number n of classes, each one characterized by its average income, the average incomes being the positive numbers $r_1 < r_2 < \cdots < r_n$. We refer to [1] for a detailed illustration of the stylized micro scale mechanism we have in mind. Here, we just recall that also the part of the government (which of course plays a role in connection with the taxation system) can be described through monetary exchanges between pairs of individuals, and we emphasise that consequently two kinds of interactions may take place: the so called *direct* ones, between an h-individual and a k-individual, occurring when the first one pays the second one, and the *indirect* ones, between the h-individual and every j-individual with $j \neq n$, occurring on the occasion of the direct h–k interaction. The indirect interactions represent the transactions corresponding to the payment of taxes and to the benefit of the redistribution. In short, and we are referring here to a tax compliance case, in correspondence to any direct h–k interaction, if S (with $S < (r_{i+1} - r_i)$

[1] The term *econophysics* was coined by H.E. Stanley.