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Efficiency of the plate-number-based traffic rationing in general networks

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ABSTRACT

Road space rationing based on vehicle plate numbers restricts vehicle access to a network based upon the license number on pre-established days. It has been used in some large cities especially when there are some major events. This paper analyzes the efficiency of road space rationing schemes by establishing the bounds of the reduction in the system cost associated with the restricted flow pattern at user equilibrium in comparison with the system cost at the original user equilibrium. The bounds are established under the general traffic equilibrium model formulated as variational inequalities and illustrated with a simple example.

1. Introduction

In a transportation network, travelers are oblivious to the delays imposed on those following them under dense traffic conditions, i.e., they seek routes that they perceive as having the least travel time from their origin to destination, under the prevailing traffic conditions. In other words, travelers minimize their individual travel times, where travel time of a route is the sum of the travel time of the links in the route. The resulting flow is called a Wardrop user equilibrium (UE), which is the situation that travelers cannot reduce their journey times by unilaterally choosing another route (Wardrop, 1952). Such selfish route-choosing behavior may lead to congestion or low efficiency of road usage, since the user equilibrium may deviate from the system optimum, i.e., the network state that the total travel time is minimized.

To alleviate road congestion and to improve the efficiency of traffic networks, travel demand management strategies through road space rationing and pricing have been proposed and studied (Daganzo, 1995; Gentile et al., 2005; May, 1986; Verhoef et al., 1997; Viegas, 2001; Wang et al., 2007). By artificially restricting demand (vehicle travel) via rationing the scarce common good road capacity during the peak periods, it is aimed to reduce the negative externalities generated by peak urban travel demand in excess of available supply or road capacity. This objective is achieved through restricting all private cars' access to a road network based upon the last digits of the license number on pre-established days and during certain periods, usually, the peak hours.

On July 20, 2008, Beijing implemented a temporary road space rationing based on plate numbers in order to significantly improve air quality in the city during the 2008 Summer Olympics (BBC, 2008). Enforcement was made with automated traffic surveillance network. The rationing was in effect for 2 months, between July 20 and September 20, for the Olympics and the following Paralympics. The restrictions on car use were in place on alternate days depending on the plates ending in odd or even numbers. This measure took 45% of the 3.3 million car fleet off the streets (Wade, 2008).[1] The 2012 Summer

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[1] Theoretically, it should reduce the number of vehicles by 50%. This is probably because a number of vehicle categories were exempt (http://www.bjjtgl.gov.cn/publish/portal0/tab91/info5504.htm). However, according to one source "the restrictions, along with an earlier ban on the use of vehicles which failed emission standards, drove about two-thirds of Beijing's 3.29 million cars off the road" (http://www.chinadaily.com.cn/olympics/2008-08/27/content_6974321.htm.

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Olympics organization, with support from the Mayor of London office, announced in 2007 that they are planning auto exclusion zones around all venues, including London, Birmingham, Manchester, Newcastle upon Tyne, Glasgow and Cardiff (Mayor of London official website). London authorities hope this measure will work as an experiment to change the public's travel behavior, allowing thereafter a shift from automobile to mass transit or bicycling. This severe policy has been publicized as the "First Car-free Olympics".

Long term implementation of road space rationing is common in Latin America, and in many cases, the road rationing has as a main goal the reduction of air pollution, such as the cases of Mexico City, and Santiago, Chile. Säo Paulo, with a fleet of 6 million vehicles in 2007, is the largest metropolis in the world with such a travel restriction, implemented first in 1996 as a measure to mitigate air pollution, and thereafter made permanent in 1997 to relieve traffic congestion. More recent implementations in Costa Rica and Honduras have had the objective of reducing oil consumption, due to the high impact this import has on the economy of small countries, and considering the steep increases in oil prices that began in 2003.

In this paper, we analyze the efficiency of road space rationing via bounding the ratio between the system cost under the new user equilibrium for given traffic demand with road space rationing and that under the original user equilibrium. There are some recent literature on efficiency analysis of the toll-based traffic management, which bounded the price of anarchy, i.e., the ratio between the system cost at UE and the minimum system cost (Chau and Sim, 2003; Cole et al., 2003a,b; Correa et al., 2004; Czumaj and Vöcking, 2002;Han et al., 2010, 2008a,b; Han and Yang, 2008; Koutsoupias and Papadimitriou, 1999; Roughgarden and Tardos, 2002; Roughgarden, 2002). To our best knowledge, this paper is the first trial in bounding the efficiency of the plate-number-based traffic rationing.

The paper is organized as follows. In the next section, we introduce some basic definitions and describe the traffic equilibrium model under consideration. In Section 3, we establish a general lower and upper bound of the system cost associated with the rationed traffic equilibrium, which depends on solutions of semi-infinite semidefinite programming problems. In Section 4, we provide concrete bounds for some special cases. In particular, when the Jacobian matrix of link cost functions is positive definite, we show that the efficiency bound can be obtained via explicitly solving a nonlinear program with two variables. We also investigate the simplified cases with asymmetric or symmetric linear link cost functions and the case with separable nonlinear link cost functions. A simple example is given in Section 5 to illustrate our results. Section 6 offers some concluding remarks.

2. Preliminaries

Let $G = (V,E)$ be a directed network, where V is the set of vertexes, and E is the set of edges. We allow parallel edges but no self-loops. Suppose that the number of users sharing the network is sufficiently large so that each user's behavior has infinitesimal effect on the other users. Let $W = \{(s,t)—s,t \in V\}$ denote the set of origin–destination (OD) pairs. For any OD pair $w = (s,t)$, let P_w denote the set of paths connecting s and t. Let $P = \bigcup_{w\in W} P_w$ denote all paths. For each edge $a \in E$, there is an edge flow f_a, which is the sum of the path flow using the edge. Let $\delta_{p,a} = 1$, if path $p \in P$ traverses edge a, and $\delta_{p,a} = 0$ otherwise. Let Δ be the edge-path incidence matrix, with ap-th entry being $\delta_{p,a}$. Let x be the path flow vector and f be the edge flow vector. We then have

$$f = \Delta x, \quad x \geqslant 0.$$

Assume there is a fixed amount of OD demand d_w for each OD pair $w \in W$. This fixed demand assumption is reasonable when the road space rationing policy is introduced temporarily for some special events such as Summer Olympics. Let Γ denote the path-OD pair incidence matrix, with entry $\gamma_{wp} = 1$ if path p connecting OD pair w, and $\gamma_{wp} = 0$ otherwise. Then, it holds

$$d = \Gamma x, \quad x \geqslant 0,$$

where d is the vector of demands.

We assign an edge cost function $c_e(f)$ to each edge $e \in E$ and let $c(f)$ denote the vector of edge cost functions.[2] The function $c(f)$ is said to be symmetric if $\partial c_i(f)/\partial f_j = \partial c_j(f)/\partial f_i$, $\forall i,j \in E$. If $c(f) = (c_i(f_i))_{i\in E}^{\top}$, i.e., c_i only depends on the flow on i for all $i \in E$, then $c(f)$ is said to be separable. The cost function $c(f)$ is said to be linear if $c(f) = Mf + \phi$, where M is an $|E| \times |E|$ matrix with $m_{i,j} \geqslant 0$, and ϕ is a vector with $\phi_j \geqslant 0$, $\forall i,j \in E$. We assume that the model is additive in the sense that for any path $p \in P$, the path cost is

$$t_p(x) = \sum_{e\in E} \delta_{p,e} c_e(f).$$

Let

$$t(x) = [t_p(x)]_{p\in P}$$

be the vector of path travel time functions. Throughout the paper, we assume that the path cost function t is monotone, i.e.,

$$(t(f) - t(g))^{\top}(f - g) \geqslant 0, \quad \forall f, \quad g \in R^{|P|},$$

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[2] During the Beijing Olympic Game period, additional traffic lanes are excluded for Olympic use, thus the capacity for these roads is changed. However, in our study, such variation of capacity for part of the network is not considered, so the edge cost function for each edge does not change before and after rationing.