European Journal of Operational Research 247 (2015) 872–878

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

A linearized value-at-risk model with transaction costs and short selling

Jing-Rung Yu [a], Wan-Jiun Paul Chiou [b,c,*], Da-Ren Mu [a]

[a] Department of Information Management, National Chi Nan University, Taiwan
[b] Finance Group, Northeastern University, USA
[c] Department of Finance and Law, Central Michigan University, USA

ARTICLE INFO

Article history:
Received 28 August 2014
Accepted 11 June 2015
Available online 18 June 2015

Keywords:
VaR
Linearized model
Short-sales
Transaction costs

ABSTRACT

Though value-at-risk (VaR) has been widely applied by finance industry, a possibility of multiple optimums that is caused by the nonlinearity in modeling can challenge its application. To deal with this technical issue, we propose a linearized VaR model that employs the mixed 0-1 programing in Lin (2009) P4 model. We further advance the previous models with considering various transaction costs and the optimization of short-selling weights. We compare the performance of buy-and-hold (BH) strategy, the mean-variance (MV) model, the original P4 model, and our linearized P4 (LP4) model by rebalancing a wide scale of international and alternative investments during a period between 2001 and 2012. The results of numerical tests show the superior performance of the VaR models to the BH and the MV portfolios. The LP4 model yields the global optimum and outperforms the corresponding P4 model in both return and risk. The stability of portfolio value generated from the LP4 model supports its higher effectiveness in risk management than the P4 model.

1. Introduction

How to construct an optimal asset allocation is a critical issue to financial industry. The usefulness of Markowitz (1952) model in the real world has been questioned due to commonly found asymmetric distribution, risk clustering, and fat tail of security returns. The need of improvement of modeling to better manage risk and portfolio comes from financial institutions, regulators, and academia. Among the alternative models, value-at-risk (VaR) has become one of popular replacements for variance to measure downside risk (Jorion, 2001). But VaR is challenged by its computational complexity, particularly its non-linear property. To alleviate this issue, Benati and Rizzi (2007) propose a mixed integer linear model to form the optimal mean-VaR portfolio (P2 or P3 model). Lin (2009) points out that their argument is not completely accurate and further suggests an alternative (P4) model. These studies yet leave some challenges affecting feasibility of portfolio model: How to deal with the nonlinearity in calculating VaR? How to improve the feasibility of VaR model by synthesizing other techniques without sacrificing the desired natures of risk management? How to model transaction costs in rebalancing portfolio over time? In this paper, we propose a linearized VaR model by improving Lin (2009) and, further in the numerical tests, apply it in managing a portfolio of a wide scale of international equities and alternative investments. We also consider allowing short-sales in asset allocation to enhance flexibility of portfolio management.

Our study responds the call of risk management in academia and on Wall Street. Though VaR model has been regarded as the international standards for financial industry and regulators, its non-linear and non-tractable properties challenge its application (Natarajan, Pachamanova, & Sim, 2008; Zymler, Kuhn, & Rustem, 2013). Benati and Rizzi (2007) suggest mixed integer linear programming VaR models. They argue that the decision maker can either seek to maximize the expected portfolio return (P2 model) or to minimize the confidence level, α^{VaR}, given r^{VaR} and required return r^* (P3 model). Benati and Rizzi (2007) conclude that the two models are equivalent. However, Lin (2009) indicates that their claim may be questionable due to the discrete values of that are caused by a finite number of observed returns in the real world. Lin (2009) further develops the P4 model, which is nonlinear and may yield multiple optimums.

Our paper contributes to the literature in three aspects. First, we propose a linearized P4 (LP4) model by applying the linearization method of mixed 0-1 polynomial programs suggested by Chang and Chang (2000). The P4 model by Lin (2009) may generate multiple optimums due to its non-linear property. Our modification in contrast ensures the solution is the global optimum and always yields an expected return not less than that of the P4 model. We provide an effective methodology in calculating the VaR of a portfolio.

Second, our study incorporates the transaction costs that are based on the market practices. The strategies without considering

* Corresponding author at: Finance Group, Northeastern University, USA.
Tel.: +1 6173738374.
E-mail addresses: jennifer@ncnu.edu.tw (J.-R. Yu), w.chiou@neu.edu (W.-J.P. Chiou), s100213542@ncnu.edu.tw (D.-R. Mu).

http://dx.doi.org/10.1016/j.ejor.2015.06.024
0377-2217/

transaction costs hinder the effectiveness of the portfolio models in the financial industry. Atkinson and Mokkhavesa (2004) suggest that the portfolio rebalancing frequencies and scale will be larger than what they should be if the transaction costs are ignored in modeling. Though overlooking trading costs can simplify an analysis, it overestimates the expected return and weakens the feasibility of the model. In this paper, we apply the methodology suggested by Fusai and Luciano (2001), Woodside-Oriakhi, Lucas, and Beasley (2013), and Kolm, Tütüncü, and Fabozzi (2014) in rebalancing portfolio with transaction costs. Our empirical findings can be useful to asset management.

Third, our study also considers optimal asset short-sales in the analysis. From a perspective of practitioners, White (1990) and Angel, Christophe, and Ferri (2003) suggests that short-selling provide an opportunity of speculation and arbitrage to investors and also helps lower portfolio risks during market downturn. Yu and Lee (2011) suggest that the weights of short selling should be optimized to avoid high risks. A reasonable design of portfolio short-sales increases the effectiveness of diversification across assets and hedging over time. We model both the costs and the weights of asset short-sales and repurchase of short-sales to enhance the flexibility in portfolio decision.

Recent studies have proposed some new methods regarding the VaR application but do not deal with its non-linearity. Fusai and Luciano (2001) find the estimate of loss using dynamic VaR is lower than that generated by the conventional VaR model. Engle and Manganelli (2004) and Ausín, Galeano, and Ghosh (2014) incorporate econometric method, such as conditional autoregressive models in assessing risk. Adrian and Brunnermeier (2011) measure the systemic risk of financial institutions by computing the difference of conditional VaR (ΔCoVaR) under different economic states. Goh, Lim, Sim, and Zhang (2014) propose a partitioned VaR (PVaR) model using half-space statistical information and find it outperforms the Markowitz model in the risk-return tradeoff. The interest of the above papers, however, is not to resolve the nonlinear issue of VaR models. In addition, the previous literature does not consider the factors that may impact feasibility of modeling, such as portfolio short-sale and trading costs. We model portfolio with incorporating the above factors to ensure the feasibility of the results.

Our empirical results using the data of international and alternative assets support the superiority of our proposed LP4 model to other portfolio strategies, including the buy-and-hold (BH), the mean-variance (MV), and the original P4 models, in managing portfolio and risk. Specifically, the LP4 model yields a higher return (r^{VaR}), higher mean-variance efficiency, and lower volatility. Though the LP4 model does not consistently realize higher market value, its realized return is less volatile than the P4 portfolio. This leads to a higher Sharpe ratio of realized return and higher effectiveness in portfolio risk management.

The structure of the rest of the paper is as follows. Section 2 presents the P2 and P4 models. Section 3 describes our proposed model. Section 4 reports the data. Section 5 demonstrates the empirical results and compares with several portfolio strategies. Section 6 concludes.

2. The P2 and P4 models

The mean-variance (MV) model by Markowitz (1952) serves as the foundation for modern portfolio theory. In light of financial crises in the past two decades have challenged the use of variance from historical data, value-at-risk (VaR) has been adopted by financial industry and regulators to replace variance as a measure of risk. According to Jorion (2001), VaR can be expressed as

$$\Pr(W_t \geq \text{VaR}) \leq 1 - \alpha$$

where W_t is the value of loss of the investments portfolio, VaR is the maximum loss, and α is the confidence level between 0 and 1.

Benati and Rizzi (2007) replace VaR as the measure of portfolio risk in the MV framework. Assuming there are n assets in an investment set, the decision vector $w = (w_1, \ldots, w_n)$ represents portfolio weights, which can be selected from all possible sets W in space $\Re^n$. Let x the random variable of portfolio return, a random vector X represents profit of portfolio in a space $\Re^N$ with N uncertain states, and F the distribution function of X. For the profit of investment portfolio at time t, the probability that the cumulative profit during a given period exceeds the VaR is α. Specifically,

$$r_\alpha^{VaR}(X) = \inf\{x | F_X(x) \geq \alpha\}$$

We start by revisiting the P2 model and then extend the P4 model to the linearized P4 (LP4) model, which is an improvement of the P2 model, by linearizing its constraints to form the optimal portfolio. The proposed LP4 model further considers asset short-sales and trading costs in portfolio modeling and rebalancing to ensure its feasibility.

2.1. The mixed integer nonlinear programming P2 model

Benati and Rizzi (2007) replace variance as VaR in the Markowitz model and apply a mixed integer nonlinear programming. They claim that the two counterpart models, Max Return/Fixed Risk problem (P2) and Min Risk/Fixed Return problem (P3), are equivalent. The P2 model suggested by Benati and Rizzi (2007) is:

$$\max \quad \sum_{t=1}^{T} \frac{1}{T} x_t \tag{1}$$

$$\text{s.t.} \quad x_t = \sum_{j=1}^{n} w_j r_{jt}, \quad t = 1, \ldots, T, \tag{2}$$

$$x_t \geq r^{Min} + (r^{VaR} - r^{Min}) y_t, \quad t = 1, \ldots, T, \tag{3}$$

$$\sum_{t=1}^{T} \frac{1}{T}(1 - y_t) \leq \alpha^{VaR}, \tag{4}$$

$$\sum_{j=1}^{n} w_j = 1, \tag{5}$$

$$w_j \geq 0, \quad j = 1, \ldots, n, \tag{6}$$

$$y_t \in \{0, 1\}, \quad t = 1, \ldots, T, \quad \text{and} \quad \begin{cases} y_t = 0, & x_t \geq r^{Min} \\ y_t = 1, & x_t \geq r^{VaR} \end{cases}, \tag{7}$$

where x_t is the random variable of the portfolio return on day t; T is the ending day; r^{Min} is the minimum return of all investments during a period of T days; w_j is the portfolio weight of asset j; r^{VaR} is the threshold return set by the investor; α^{VaR} is the confidence level between 0 and 1. Constraint (2) sets x_t as the linear combination of r_{jt}, and (3) and (4) prevent to select the portfolios that yield VaR lower than the fixed threshold. Furthermore, (5) and (6) set the budget constraint and prohibit short selling, and (7) shows the binary variable y_t indicating whether return on tth day is lower than r^{VaR} at the probability α^{VaR}.

2.2. The P4 model

Lin (2009) indicates that the P2 and P3 models in Benati and Rizzi (2007) are partly equivalent since the P3 model is not to minimize VaR but to minimize the probability that the portfolio return lower the threshold. In addition, it is questionable to set the r^{VaR} *a priori* and to assume continuity of returns in the real-world. Lin (2009) further develops the P4 model:

$$\max \quad r^{VaR} \tag{8}$$

$$\text{s.t.} \quad \sum_{t=1}^{T} \frac{1}{T} x_t \geq E, \tag{9}$$