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Hysteresis bands on returns, holding period and transaction costs ☆

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ABSTRACT

In the presence of transactions costs, no matter how small, arbitrage activity does not necessarily render equal all riskless rates of return. When two such rates follow stochastic processes, it is not optimal immediately to arbitrage out any discrepancy that arises between them. The reason is that immediate arbitrage would induce a definite expenditure of transactions costs whereas, without arbitrage intervention, there exists some, perhaps sufficient, probability that these two interest rates will come back together without any costs having been incurred. Hence, one can surmise that at equilibrium the financial market will permit the coexistence of two riskless rates that are not equal to each other. For analogous reasons, randomly fluctuating expected rates of return on risky assets will be allowed to differ even after correction for risk, leading to important violations of the Capital Asset Pricing Model. The combination of randomness in expected rates of return and proportional transactions costs is a serious blow to existing frictionless pricing models.

1. Introduction

Investors, who have to pay transactions costs, optimally rebalance their portfolio at points in times that are random and are not easily observable. Instead, the financial econometrician measures rates of return on financial assets over regular, fixed intervals in time. Investors compare the rates of return on assets over the forthcoming holding periods while the econometrician testing the validity of an asset pricing model, arbitrarily attempts to compare them over successive weeks, months or years.

We would like to know whether it is possible meaningfully to compare the rates of return on two otherwise similar assets when the rates are measured at regular intervals, while investors trade at random times. The question cannot be addressed without a model of the way in which investors choose to rebalance or not their portfolios. We first consider the case of two riskless assets in a portfolio. Then we extend the analysis to risky, long-lived assets such as equities.

If two interest rates on deposits were to remain unequal forever, it would pay to arbitrage out their difference immediately, even if transactions costs had to be incurred in doing so. In the absence of discounting, and in the absence of any costs for rolling over the deposits, the interest differential earned by the arbitrage would eventually outweigh any finite transactions costs incurred at the outset of the arbitrage operation.

If, however, the spread between the two rates fluctuates randomly, it may no longer pay to start an arbitrage. The interest differential may not last long enough to cover profitably the transactions costs. This basic idea was put forth originally in Baldwin (1990) who argues that very small transactions costs help in accounting for the failure of foreign exchange market efficiency tests and shows that the problem mathematically resembled Dixit (1989) problem of stochastic entry and exit.[1]

The purpose of the present paper is to re-formulate this idea of no-arbitrage spread between the rates of return on two different types of assets and exploit it in the context of an optimal portfolio choice problem with transactions costs. We first examine the

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[1] See also Baldwin and Lyons (1994).

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portfolio choice of an investor with given relative risk aversion who has access to two riskless investments with instantaneous returns (infinitesimal maturity). One of these brings a rate of interest that is constant over time while the other yields a rate that varies according to a stochastic process. The process incorporates a reversion force, which in the long run pulls the second rate towards the first one. We approach this problem of portfolio choice in the manner of Dumas and Luciano (1991), postponing final consumption to a point infinitely into the future, and computing the stationary optimal policy. We consider two cases: in the first, the investor is restricted to hold all his wealth in the form of one asset or the other, whereas in the second the asset holding are allowed to vary continuously. We start with the all-or-nothing case, in which we have an analytical solution, because it delivers the intuition of the result, then we show that roughly the same result, with the same intuition, holds when the portfolio choice is not restricted; finally we show the implications for the more common case situation in which there exists a risky security.

For a given portfolio imbalance, the investors allow some gap between the two rates to survive; this gap is called "the hysteresis band". We are interested in the size of this gap. We intend to show that the gap is much larger than the transactions costs. Because deposits are not forcibly refunded and can be rolled over costlessly, the period over which a given investor holds the deposit – the "holding period" – is a decision variable.[2] As smaller and smaller transactions costs are considered, the allowable gap (or spread) measured over the holding period is gradually compressed but the anticipated optimal holding period shrinks because smaller transactions make it less costly to switch from one asset to the other. Depending on the rates at which these two variables approach zero, the allowable annualized quoted spread may become small slowly or quickly. We show that it becomes small at a cubic-root rate.

It is important to underline that the analysis corresponding to the riskless investments is meant to show that *non-IIDness of returns can be sufficient to create a gap*. We use the simplest menu of assets to make the point. In addition, even with two riskless securities, the arbitrage is not riskless when there are transactions costs.

Later on, we consider an arbitrage between a riskless asset with a constant rate and a risky asset with a stochastic mean-reverting conditionally expected rate of return. We find that, as transaction costs approach zero, the size of the hysteresis bands converges to zero at a slower pace and we conclude that the CAPM must be badly violated because of the existence of transactions costs. Moreover, our model is able to generate an expected time from purchase to sell one order of magnitude smaller than the holding period shown in Constantinides (1986). The difference in the two results is traceable to the difference in the assumed behavior of the conditionally expected return on the risky asset. Constantinides considers an expected return that is constant; we consider a stochastic, mean reverting one.

Mean reversion in expected stock returns has been first studied empirically by Fama and French (1988), Poterba and Summers (1988), and Bekaert and Hodrick (1992) among others.[3] We contribute to the asset-allocation literature solving a portfolio-choice problem with transaction costs and mean-reverting expected returns. In this sense, we extend the works of Davis and Norman (1990), Dumas and Luciano (1991), Akian et al. (1996), Eastham and Hastings (1988), Liu (2004) who determine the optimal portfolio policy in case of proportional transaction costs and constant investment set, and Kim and Omberg (1996), Campbell and Viceira (1999), and Wachter (2002) who instead consider mean reverting expected returns but no transaction costs.

Shreve et al. (1991) investigate the problem of optimal consumption and investment of a agent trading in two bonds with constant rates of return and facing both transaction costs and solvency constraints. In their model, both types of frictions are necessary to limit the ability of the investor to arbitrage the two assets. In a setting with time-varying investment opportunities and trading costs, Lynch and Balduzzi (2000) show that predictability calibrated to U.S. data continues to have a large effect on the rebalancing behavior of a multiperiod investor. Grinold (2006) derives the optimal steady-state position with quadratic trading costs and a single predictor of stock returns per security. Jang et al. (2007) propose a regime-switching model of portfolio choice with transaction costs and show that jumps in regime, by entailing time-varying investment opportunity set, generate first-order effects on liquidity premia. More recently, Gârleanu and Pedersen (2013) investigate a dynamic-portfolio choice problem with transitory and persistent transaction costs in which stock returns are driven by multiple predictors with different mean-reversion speeds.

Dai et al. (2011) study the optimal investment problem of a mutual fund that faces position limits and trades a risk-free asset, a liquid stock, and an illiquid stock that is subject to proportional transaction costs. They assume a constant investment opportunity set and a finite horizon economy, finding that (i) the buy and sell boundaries are time-varying and (ii) the optimal trading strategy is non-myopic with respect to position limits because position limits will for sure bind when time to horizon becomes short enough. In our paper, first we depart from the constant investment opportunity set and, second, we are interested in determining the stationary (or infinite-horizon) trading strategy in the presence of proportional transaction costs. In our "infinite-horizon economy", the buy and sell boundaries are not affected by future, currently not binding portfolio constraints, since the time horizon never becomes small enough and, in any case, we consider a logarithmic investor who exhibits myopic behavior.

Lynch and Tan (2011) investigate a life-cycle (that is a finite-horizon) portfolio choice problem with transaction costs, short-selling constraints and a variety of more elaborate settings, such as return predictability, wealth shocks and state-dependent transaction costs. Again, they find time-varying trading boundaries. We investigate the optimal trading barriers of an infinitely-lived investor, which is not always constrained in his portfolio composition, finding that the no-transaction region is time-invariant. Finally, Bacchetta and van Wincoop (2010) contribute to this literature by examining the impact of infrequent portfolio decisions on the forward discount puzzle. They show that asset management costs discourage investors from active trade, accounting for large deviations from the uncovered interest parity.

The paper is organized as follows. In Section 2 we solve the basic portfolio problem considered by Baldwin (1990) in which investors are constrained to investing their entire wealth in one riskless asset or the other; we measure the resulting gap in interest rates. In Section 3, we allow continuous adjustment of the portfolio while still considering only two riskless assets. In Section 4, we optimize a portfolio made up of one riskless asset with a constant rate and one risky asset with a mean reverting expected return. Section 5 concludes. Finally, the appendixes contains technical details and some additional results not reported in the main text.

[2] The analysis is not limited to bank deposits. In fact, it applies to all assets. Shares of stock that pay no dividend are automatically "rolled over" until the investor explicitly sells them. Section 4 will be devoted to the analysis of rates of returns on equities. The analysis could, but will not, be generalized to shares that pay a dividend. Bonds would require a separate study because they are 100% refunded at the maturity date. That is one "transaction" that is forced on the bondholder.

[3] In particular, Fama and French have shown that long-holding period returns display mean reversion. The behavior of long-period returns is the combined result of short-period mean behavior and volatility behavior. In our model, short-period volatility is assumed constant.