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Slack free MEA and RDM with comprehensive efficiency measures

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ABSTRACT

This paper extends two directional distance function models, the Multi-directional Efficiency Analysis (MEA) Model and the Range Directional Model (RDM), in order to account for any type of technical inefficiency, i.e. both directional and non-directional inefficiencies. We first focus on the variable returns to scale (VRS) case, because both VRS-MEA and RDM are translation invariant models, which mean that both models are able to deal with negative data. Our main result is the definition of a new comprehensive efficiency measure which is units invariant and translation invariant and covers both models. Secondly, we introduce the RDM model under constant returns to scale (CRS) together with a new comprehensive efficiency measure.

1. Introduction

It is well known that while the radial DEA models [1,2] can be used to detect both radial and non-radial inefficiencies, the actual DEA efficiency score only represents the radial efficiency. Consequently, any non-radial slack, or mix inefficiency, is not accounted for by the DEA score and the DEA score is not a comprehensive efficiency measure since it does not account for all forms of technical efficiency. A similar point can be made for directional distance function models, such as the Range Directional Model (RDM) of Silva Portela et al. [3], where the directional efficiency score does not account for any non-directional slack. The Multi-directional Efficiency Analysis (MEA) Model of Bogetoft and Hougaard [4], is also directional in nature, but the directional efficiency measure must be derived ex post and is furthermore based on a benchmark selection that may contain non-directional slack.

The first proposed comprehensive efficiency measure was the Russell Graph Measure of Färe and Lovell [5]. While this remains a theoretical contribution it has still inspired most of the subsequently developed comprehensive efficiency measures. For instance, several DEA efficiency measures have been suggested based on the additive model [6], which detect all the technical inefficiency in all dimensions such that the benchmarks lie on the strongly efficient frontier and do not have any slack (c.f. Lovell et al. [7], Cooper et al. [8], Pastor et al. [9] and Tone [10]). None of these models do, however, consider a directional vector for the projections onto the efficient frontier, which constitutes an alternative approach with virtues argued by, e.g. Fare and Grosskopf [11]. The existing comprehensive efficiency measures are based on the same basic idea: The objective functions of their corresponding linear programming problems measure the efficiency of each unit based on the absolute inefficiencies in each dimension (often also denoted total slacks) rather than based directly on the actual projection (or benchmark). Alternatively, Silva-Portela and Thanassoulis [12] propose to first identify a projection onto the efficient frontier and secondly measure the inefficiency in all dimensions by comparing the unit under analysis to its benchmark. Unfortunately, the corresponding efficiency measure may not be identical for all alternative optimal projections which is required in order for the measure to be well defined (cf. the criteria for efficiency measures of Cooper et al. [8]).

Similar considerations underlie two somewhat related directional distance function models: The Multi-directional Efficiency Analysis (MEA) model of Bogetoft and Hougaard [4] and the Range Directional Model (RDM) of Silva Portela et al. [3]. Both these models consider, for each unit under analysis, an ideal point that determines the direction towards the efficient frontier where the benchmark must be located. There are, however, three main differences between the MEA and RDM models: firstly, while RDM considers the same ideal point for all units, MEA selects a specific ideal point for each unit. Secondly, while RDM directly provides an efficiency measure together with a benchmark, MEA mainly provides a benchmark selection. In MEA, an efficiency measure is

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easily derived "a posteriori", but requires the imposition of certain normalisation conditions. And thirdly, while RDM specifically assumes variable returns to scale (VRS), MEA can be defined under any returns to scale assumption. In Section 5 of this paper we will, however, show how RDM can be extended to the case of constant returns to scale.

Before proceeding it is worth noting how the ideal points are determined in RDM and MEA, respectively: The unique ideal point of RDM is the "zenith" of the observed data set, with each input value being the minimum of the observed data values for this input and each output value the maximum of the data values for that output. MEA, on the other hand, defines an ideal point specific to each unit under analysis. In an input-oriented analysis, the largest reduction potentials are identified in each input separately and provide the minimum possible input usage in each dimension, which are then combined to construct the ideal point. While all previous empirical applications of MEA have been input-oriented [13–15], the approach is easily modified to the output-oriented case, and we here also propose the more general case of considering input reductions and output augmentations simultaneously.

After determining the ideal point(s) in either RDM or MEA, the directional vector for each unit under analysis is given by the difference between the unit under analysis and the ideal point. Subsequently, the linear programming problem for the corresponding directional distance function is solved in order to obtain the benchmark, which is the projection onto the efficient frontier. As illustrated in Appendix A, these projections do, however, not always belong to the strongly efficient frontier. Consequently, the efficiency measure provided by RDM and the one derived through MEA are not necessarily comprehensive measures. To overcome this problem we here propose the use of second phase models that ensure slack free benchmarks, and based on these provide new comprehensive efficiency measures for VRS-MEA and RDM.

Finally, note that VRS-MEA and RDM share a nice property in being affine invariant under VRS, as shown in the two original papers. That means that they are both units invariant and translation invariant. While units invariance is an economical must, translation invariance broadens the scope of application of these models, such as enabling analysis of data sets containing negative values [16–18].

The rest of this paper unfolds as follows. In Section 2 we propose a new comprehensive efficiency measure for RDM and relate it to the directional RDM efficiency measure. In Section 3 the same new efficiency measure is applied to VRS-MEA. Section 4 provides an empirical illustration for the VRS case. Section 5 considers, for the first time, the CRS-RDM model and finally Section 6 concludes the paper.

2. The RDM model and a corresponding comprehensive efficiency measure

The RDM model was originally defined under VRS for one basic reason: the ability to evaluate datasets containing negative data. In other words, it is a translation invariant model. Furthermore, the special case where the RDM ideal point belongs to the data sample is excluded. Otherwise, the efficiency frontier will be determined by a single point like any Leontieff frontier and the RDM efficiency measure, besides not being defined for this point, will also take a value of 1 for all other units since their projections will be exactly the ideal point. Hence, regardless of the returns to scale assumption, we know that any unit in the observed data set can be connected to the RDM ideal point by a straight line that (starts at or) intersects the efficient frontier.

Letting $(\mathbf{x}_0, \mathbf{y}_0) \in \Re_+^{m+s}$ denote the unit under analysis amongst n observed production units, j (j=1,…,n), consuming x_{ji} of each of m inputs (i=1,…,m) to produce s outputs y_{jr} (r=1,…,s), the RDM model of Silva Portela et al. [3] is defined as

$$\begin{aligned}
&\text{Max}\,\beta_{RDM}\\
&\text{s.t.}\\
&\quad\sum_{j=1}^{n}\lambda_j y_{jr} \ge y_{0r} + \beta_{RDM}R_{0r}^{+}, \quad r=1,\ldots,s,\\
&\quad\sum_{j=1}^{n}\lambda_j x_{ji} \le x_{0i} - \beta_{RDM}R_{0i}^{-}, \quad i=1,\ldots,m,\\
&\quad\sum_{j=1}^{n}\lambda_j = 1,\\
&\quad\lambda_j \ge 0, \quad j=1,\ldots,n,
\end{aligned}\tag{1}$$

where

$$R_{0r}^{+} = \overline{y}_r - y_{0r}, \quad r=1,\ldots,s,$$

$$R_{0i}^{-} = x_{0i} - \underline{x}_i, \quad i=1,\ldots,m,$$

$$\overline{y}_r = \max_j\{y_{jr}\}, \quad r=1,\ldots,s,$$

$$\underline{x}_i = \min_j\{x_{ji}\}, \quad i=1,\ldots,m.$$

Regardless of whether or not the peer units chosen by the RDM model (1) have non-directional slack, the projection determined by the directional vector may not belong to the strongly efficient frontier. And if weakly efficient units are used as benchmarks, the true amount of slack (compared to the strongly efficient frontier) is furthermore not evident from (1) at all. Ensuring that only strongly efficient benchmarks are selected requires the use of a second phase procedure, typically performed using an additive model[1] [6]. With the aim of subsequently defining a new comprehensive efficiency measure we first propose to consider the following second phase weighted additive model:

$$\begin{aligned}
&\text{Max}\left(\sum_{i=1}^{m}\frac{\tau_{0i}^{-}}{R_{0i}^{-}} + \sum_{r=1}^{s}\frac{\tau_{0r}^{+}}{R_{0r}^{+}}\right)\\
&\quad\text{s.t.}\\
&\quad\sum_{j=1}^{n}\lambda_j y_{jr} - \tau_{0r}^{+} = y_{0r} + \beta_{RDM}^{*}R_{0r}^{+}, \quad r=1,\ldots,s,\\
&\quad\sum_{j=1}^{n}\lambda_j x_{ji} + \tau_{0i}^{-} = x_{0i} - \beta_{RDM}^{*}R_{0i}^{-}, \quad i=1,\ldots,m,\\
&\quad\sum_{j=1}^{n}\lambda_j = 1,\\
&\quad\lambda_j \ge 0, \quad j=1,\ldots,n,
\end{aligned}\tag{2}$$

where β_{RDM}^{*} is the optimal solution to (1) above and specific to $(\mathbf{x}_0, \mathbf{y}_0)$ though we to simplify the notation suppress the subscript $_0$ here.

Letting τ_{0i}^{-*} and τ_{0i}^{+*} be the optimal input- and output-slacks from (2), the coordinates of the projection $(\mathbf{x}_0^*, \mathbf{y}_0^*)$ following from (2) are given as $x_{0i}^* = x_{0i} - \beta_{RDM}^{*}R_{0i}^{-} - \tau_{0i}^{-*}, i=1,\ldots,m$ and $y_{0r}^* = y_{0r} + \beta_{RDM}^{*}R_{0r}^{+} + \tau_{0r}^{+*}$, $r=1,\ldots,s$

Define the improvement ranges for the new benchmark $(\mathbf{x}_0^*, \mathbf{y}_0^*)$ as $R_{0r}^{+*} = \overline{y}_r - y_{0r}^*$, $r=1,\ldots,s$ and $R_{0i}^{-*} = x_{0i}^* - \underline{x}_i$, $i=1,\ldots,m$.

The *comprehensive efficiency measure* we are proposing for the RDM model is given by

$$\overset{\leftrightarrow}{\Gamma}{}^{*}_{RDM} = \frac{1}{m+s}\left(\sum_{i=1}^{m}\frac{R_{0i}^{-*}}{R_{0i}^{-}} + \sum_{r=1}^{s}\frac{R_{0r}^{+*}}{R_{0r}^{+}}\right).\tag{3}$$

[1] A recently quite popular additive model is the SBM model of Tone [10], c.f. e.g. Avkiran [19,20], Avkiran and Rowlands [21] and Tone and Tsutsui [22].