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Modelling security market events in continuous time: Intensity based, multivariate point process models

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Abstract

A continuous time econometric modelling framework for *multivariate* financial market event (or 'transactions') data is developed in which the model is specified via the vector conditional intensity. Generalised Hawkes models are introduced that incorporate inhibitory events and dependence between trading days. Novel omnibus specification tests based on a multivariate random time change theorem are proposed. A bivariate point process model of the timing of trades and mid-quote changes is then presented for a New York Stock Exchange stock and related to the market microstructure literature. The two-way interaction of trades and quote changes in continuous time is found to be important empirically.

JEL classification: C32; C51; C52; G10

Keywords: Point process; Conditional intensity; Hawkes process; Specification test; Random time change; Transactions data; Market microstructure

1. Introduction

This paper develops a continuous time econometric modelling framework for analysis of the dynamic microstructure of financial markets—that is, their dynamic evolution viewed at a very fine level of detail. These dynamics can be described in terms of the stochastic occurrence times and characteristics of well-defined *market events* such as trades and changes to the quoted prices. The work is motivated by the growing theoretical market

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microstructure literature and the advent of data sets providing complete records for some or, in the case of certain electronic markets, all types of market event. Such data sets potentially provide an enormous amount of information about the intraday behaviour of financial markets and allow testing of the hypotheses of the theoretical literature. However, progress in modelling the data in continuous time has hitherto been hindered by the difficulties presented by the multivariate case. Furthermore, many interesting economic questions concerning financial market microstructure can only be addressed using such multivariate models. The approach adopted here overcomes these difficulties by focusing on the *conditional intensities* of the market event arrival processes, and provides a general framework for model specification and inference that it is hoped will greatly facilitate the econometric analysis of these vast and important data sets in the future.

The main contributions of the paper may be summarised as follows. First, an intensity-based approach to model specification is used to develop a new class of models that permits the analysis of *multivariate* financial market event (or 'transactions') data—that is, data that records the timing and characteristics of several different *types* of market event.[1] The models are general enough to incorporate 'inhibitory' events that result in a decrease in a conditional intensity and to allow dependence between trading days. Second, the use of a multivariate random change of time to construct specification tests for parametric point process (PP) models is established. Novel omnibus tests for the multivariate case are proposed together with tests of the specification of each component, scalar conditional intensity. A sufficient condition for the validity of the testing procedures is derived which is shown to be natural in the context of financial markets. Finally, applying the econometric methods developed in the paper to data for a New York Stock Exchange (NYSE) stock provides evidence that the *two-way* interaction between the timing of trades and quote changes is important empirically.

The development of continuous time models for market event data is an important challenge in financial econometrics for the following reasons. First, models set in 'event time' may well ignore aspects of the evolution of the market that are economically important. Indeed, a growing number of papers point to the economic significance of real, 'wall-clock' time (see, *inter alia*, Easley and O'Hara, 1992; Hasbrouck, 1999; Dufour and Engle, 2000). Second, most potential practical applications such as volatility measurement and the design of optimal order submission strategies (see Harris, 1998) require that the models relate to real time. Finally, a standard time series analysis of aggregated data using fixed intervals of real time involves an undesirable loss of information since the characteristics and timing relations of individual events are lost.

In the econometric analysis, the market event data are viewed as the realisation of a multivariate PP: that is, as the realisation of a double sequence, $\{T_i, Z_i\}_{i\in\{1,2,\ldots\}}$, of random variables where T_i is the occurrence time of the ith event and $Z_i \in \{1, 2, \ldots, M\}$ indicates the ith event's type. Whilst considerable progress has been made in modelling the univariate ($M = 1$) case using time series models of the intervals or durations between events (see in particular, Engle and Russell 1997, 1998), multivariate extensions of this work have been slow to emerge in the econometrics literature.[2] Engle and Lunde (2003)

[1] Such data is referred to here as 'market event data' because the term 'transactions' is often taken to be synonymous with trades.

[2] Univariate models of market event data, and autoregressive conditional duration models in particular, are surveyed by Bauwens and Giot (2001).