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Equilibrium in securities markets with heterogeneous investors and unspanned income risk ☆

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Abstract

In a finite time horizon, incomplete market, continuous-time setting with dividends and investor incomes governed by arithmetic Brownian motions, we derive closed-form solutions for the equilibrium risk-free rate and stock price for an economy with finitely many heterogeneous CARA investors and unspanned income risk. In equilibrium, the Sharpe ratio is the same as in an otherwise identical complete market economy, whereas the risk-free rate is lower and, consequently, the stock price is higher. The reduction in the risk-free rate is highest when the more risk-averse investors face the largest unspanned income risk.

JEL classification: G12; G11; D53

Keywords: Unspanned income; Heterogeneous preferences; Continuous-time equilibrium; Risk-free rate puzzle; Equity premium; Incomplete markets; Brownian motion

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1. Introduction

Labor income is an important source of wealth for most individuals (see [8,28]) with a potentially large impact on their consumption and portfolio decisions and, consequently, on the equilibrium securities prices. However, because labor income typically has a large unhedgeable component, solving for the optimal decision rules and equilibrium prices is very difficult. Closed-form solutions for the optimal individual decisions with unspanned income have been derived in a few settings with either preferences for consumption over an infinite time interval or for consumption at a single future date. We derive closed-form solutions for the more appropriate case of consumption over a finite time interval for an incomplete market, continuous-time setting with negative exponential (CARA) utilities and dividends and labor income governed by arithmetic Brownian motions. More importantly, using these individually optimal decision rules, we derive the first closed-form solution in the literature for the equilibrium risk-free rate and the equilibrium stock price in a setting with a finite set of heterogeneous investors as well as unspanned income risk both at the individual and the aggregate level.

For fixed aggregate consumption dynamics, the equilibrium stock market Sharpe ratio is the same as in an otherwise identical dynamically complete market economy in which all risks are spanned, whereas the equilibrium risk-free rate (and the expected stock return) is lower and, consequently, the equilibrium stock price is higher. Our closed-form equilibrium thus explains the findings of earlier numerical studies (references are given below) that persistent non-hedgeable income shocks can help in explaining the risk-free rate puzzle.[1] Our main contribution to the literature is the closed-form equilibrium for the incomplete market, continuous-time economy. This allows us to quantify and to clearly explain the underlying economics of these earlier numerical findings.

The lower risk-free rate in our equilibrium is due to the fact that, in equilibrium, the investors evaluate their future unspanned income risk using their personal risk tolerance as opposed to the aggregate risk tolerance in the economy which they are using for evaluating risks that can be efficiently shared in the market. Hence, the risk premium for aggregate consumption risk in the determination of the equilibrium risk-free rate is higher in the setting with unspanned income risks than in settings in which these risks can be efficiently shared. Moreover, our equilibrium demonstrates the importance of the distribution of income risk across individuals: the reduction in the risk-free rate is highest when the more risk-averse investors face the largest unspanned income risk.

Our economy is formulated in a continuous-time, finite time horizon model with a single consumption good, a risk-free asset and a single risky asset. The risky asset is a claim to an exogenous dividend stream represented by an arithmetic Brownian motion. There are a finite number of consumer-investors maximizing time-additive negative exponential utility of consumption, with heterogeneity in their subjective time preference rates and in their absolute risk aversions. Each investor receives an exogenous income stream represented by another arithmetic Brownian motion, which is imperfectly correlated with the stock's dividend process and, therefore, also imperfectly correlated with the stock's price process. In other words, each investor's income process contains an unhedgeable risk component.

[1] The risk-free rate puzzle, first identified by Weil [59], refers to the observation that the historical risk-free rate is smaller than the risk-free rate predicted by simple consumption-based representative agent models. In order to match the observed equity premium in these models, the risk aversion parameter has to be very high, but then the risk-free rate becomes too high, unless we also allow for a negative time preference rate.