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Time-consistent reinsurance and investment strategies for mean–variance insurer under partial information

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HIGHLIGHTS

- Study optimal investment and reinsurance problem under partial information.
- The performance of the control problem is mean–variance utility.
- Find time-consistent equilibrium strategy in a game theoretic framework.
- Derive closed-form optimal investment–reinsurance strategy and value function.
- Compare results under partial information with complete information.

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ABSTRACT

In this paper, based on equilibrium control law proposed by Björk and Murgoci (2010), we study an optimal investment and reinsurance problem under partial information for insurer with mean–variance utility, where insurer's risk aversion varies over time. Instead of treating this time-inconsistent problem as pre-committed, we aim to find time-consistent equilibrium strategy within a game theoretic framework. In particular, proportional reinsurance, acquiring new business, investing in financial market are available in the market. The surplus process of insurer is depicted by classical Lundberg model, and the financial market consists of one risk free asset and one risky asset with unobservable Markov-modulated regime switching drift process. By using reduction technique and solving a generalized extended HJB equation, we derive closed-form time-consistent investment–reinsurance strategy and corresponding value function. Moreover, we compare results under partial information with optimal investment–reinsurance strategy when Markov chain is observable. Finally, some numerical illustrations and sensitivity analysis are provided.

1. Introduction

To control risk exposure, insurers can invest in a financial market, purchase reinsurance contract and acquire new business

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(acting as a reinsurer for other insurers). Topic about optimization problems with various objectives in insurance risk management has been extensively investigated in the literature. For example, Browne (1995) initiates study of explicit investment strategy for an insurance company to maximize expected utility of terminal wealth or minimize ruin probability with surplus process modeled by a drifted Brownian motion. Yang and Zhang (2005) study optimal investment strategies for an insurer to maximize expected exponential utility of terminal wealth or maximize survival probability with surplus process satisfied by a jump–diffusion model. Furthermore, Xu et al. (2008), Gu et al. (2010), Liang et al. (2011) and Guan and Liang (2014) study optimal investment–reinsurance policies for an insurer who maximizes expected utility of terminal wealth in different situations.

Since the introduction by Markowitz (1952), mean–variance portfolio selection problem has become a key research topic in finance, as a result, recently many scholars consider the optimal investment–reinsurance policies for insurers under mean–variance criterion. For example, Bäuerle (2005) studies optimal proportional reinsurance/new business problem and gets closed-form optimal strategy under mean–variance criterion with surplus process driven by classical Cramér Lundberg model. Bai and Zhang (2008) use linear quadratic method and dual method to derive explicit optimal investment–reinsurance policies for an insurer under mean–variance criterion, where the surplus of the insurer is depicted by Cramér Lundberg model and an approximated diffusion model. Zeng et al. (2010) assume that the surplus of an insurer is modeled by a jump–diffusion process, and use stochastic maximum principle to derive closed-form optimal investment policies under benchmark and mean–variance criteria.

However, since mean–variance criterion does not satisfy the iterated-expectation property, stochastic control problem for mean–variance criterion is time-inconsistent in the sense that Bellman optimality principle does not hold, which means a control maximizes the mean–variance utility at time zero may not be optimal for mean–variance utility at later time. Strotz (1955) proposes another way to handle time-inconsistent problem: finding time-consistent strategies instead of treating it as a pre-committed problem. Recently, Ekeland and Pirvu (2008) first provide a precise definition of equilibrium concept in continuous time. Basak and Chabakauri (2010) use dynamic programming method to derive the time-consistent solution for mean–variance problem. Björk and Murgoci (2010) study time-inconsistent problem in a general Markov framework and derive an extended HJB equation associated with verification theorem. Inspired by work as referred, Zeng et al. (cf., Zeng and Li, 2011, Zeng et al., 2013) study time-consistent optimal investment–reinsurance strategy for mean–variance insurers with asset–liability modeled by jump–diffusion process. Wei et al. (2013) get time-consistent solution to mean–variance asset–liability management problem with regime-switching.

On the other hand, regime-switching models have become popular among academia and practitioners since Hamilton (1989) pioneers the econometric applications of regime-switching models, which can reflect different market states (saying *Bullish* versus *Bearish*), see Zhou and Yin (2003). Mostly, regime switching is modeled by a continuous Markov chain, in practice, the Markov chain sometimes is not directly observable and has to be inferred from observable price processes of stocks or other assets, therefore hidden Markov model has been largely used in optimal investment problems concerning partial information. Since Lakner (1995) reclaims utility maximization with partial information, numerous work emerge. For example, Sass and Haussmann (2004) and Rieder and Bäuerle (2005) consider portfolio optimization problem with asset's drift process modeled by a unobservable continuous time Markov chain. Xiong and Zhou (2007) study mean–variance portfolio selection problem under partial information and Elliott et al. (2010) consider a similar problem under a hidden Markovian regime-switching model. More recently, Liang and Bayraktar (2014) investigate optimal investment–reinsurance problem with unobservable claim size and intensity.

Unfortunately, as far as we are concerned, little literature of insurance cares about time-consistent strategy for mean–variance insurers under partial information. But as the time-consistency of the strategy is a basic requirement of rational decision making and insurers are only accessible to partial information of the market state in many situations, in this paper, we try to consider an optimal investment and reinsurance problem for mean–variance insurers under partial information and aim to find the corresponding time-consistent strategy. In particular, we assume that the drift rate of stock is Markov-modulated and it cannot be observed directly by the insurer and the claim process suffered by insurance company is modeled by classical compound Poisson process. Also, proportional reinsurance and acquiring new business are available in insurance market. Different from Zeng and Li (2011) and Zeng et al. (2013) assuming that insurer's risk aversion level is a constant and Björk et al. (2014) assuming that insurer's risk aversion is inversely proportional to wealth, we assume that insurer's risk aversion is different for different market states and the instant risk aversion is determined by his or her estimation of the market state right now, and the estimation is made by filtering Markov chain using data of the stock's price process. To solve this partially observed optimization problem, we first transfer it to an equivalent problem with complete observations using reduction technique, and to get time-consistent investment–reinsurance strategy for this time-inconsistent mean–variance problem, we then derive an extended HJB equation and corresponding verification theorem. Finally we compare the investment–reinsurance strategy under partial information with strategy adopted if the market state can be observed directly, effects of partial information on investment–reinsurance policy are illustrated.

The paper is organized as follows: The model of insurer's wealth process with proportional reinsurance and optimization objective is presented in Section 2, moreover, definitions of equilibrium control law (time-consistent strategy) and value function are given. In Section 3 we reduce the partially observable problem to an equivalent completely observable problem and derive the generalized extended HJB equation. In Section 4 we solve the generalized HJB equation explicitly by making an Ansatz. In Section 5 we derive the equilibrium control law and the corresponding equilibrium value function given the market state is observable. Section 6 provides a sensitivity analysis to clarify the effects of several parameters on equilibrium control law and compares partially and completely observable cases. Section 7 concludes the paper.

2. The risk model

In this section we consider an insurer whose proportional reinsurance is available. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbf{P})$ is a filtered complete probability space and $\mathcal{F}_t$ is the information of the market available up to time t. $[0, T]$ is a fixed time horizon. All the processes introduced below are assumed to be adapted to $\{\mathcal{F}_t, t \in [0, T]\}$. The insurer's surplus process is modeled by classical Lundberg model

$$\mathrm{d}X(t) = c\mathrm{d}t - \mathrm{d}\left\{\sum_{i=1}^{N_t} Y_i\right\},$$

where c is premium rate of the insurer, Y_i is size of the ith claim, the total amount of claims up to time t is denoted by homogeneous Poisson process N_t with intensity $\lambda > 0$, and $N = \{N_t\}$ is independent of $\{Y_i\}$. All the claims $Y_i, \ i = 1, 2, 3, \ldots$ are assumed to be