Physica A 433 (2015) 171–181

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A quantitative description for efficient financial markets

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HIGHLIGHTS

- A feedback control model for asset price dynamics and value discovery is presented.
- Market efficiency is defined as robust asymptotic price–value equality.
- A complete characterization of the trader structure for which the market is efficient is presented in the main result of the article.
- The main result illustrates that the more transparent the market is, the more efficient it is.
- Investor rationality is not required for the results to hold true.

ARTICLE INFO

Article history:
Received 9 September 2014
Received in revised form 1 February 2015
Available online 1 April 2015

Keywords:
Efficient market hypothesis
Tâtonnenent
Internal model principle
Value discovery
Asset value dynamics

ABSTRACT

In this article we develop a control system model for describing efficient financial markets. We define the efficiency of a financial market in quantitative terms by robust asymptotic price–value equality in this model. By invoking the Internal Model Principle of robust output regulation theory we then show that under No Bubble Conditions, in the proposed model, the market is efficient if and only if the following conditions hold true: (1) the traders, as a group, can identify any mispricing in asset value (even if no one single trader can do it accurately), and (2) the traders, as a group, incorporate an internal model of the value process (again, even if no one single trader knows it). This main result of the article, which deliberately avoids the requirement for investor rationality, demonstrates, in quantitative terms, that the more transparent the markets are, the more efficient they are. An extensive example is provided to illustrate the theoretical development.

1. Introduction

1.1. Background

One of the most remarkable empirical facts of capitalist economic systems is that they appear to allocate resources efficiently in the absence of any external guidance. The "invisible hand" theory, originally set forth by Adam Smith [1], postulates that if each consumer is allowed to freely choose what to purchase and each producer is allowed to freely choose his or her product line, the market will settle on a product distribution and prices that are beneficial for the entire economy. This settlement occurs by means of a self-regulating process, the Walrasian tâtonnement (see e.g. Ref. [2]), which involves a search of balance of net supply and demand for the products based on the current observed prices for the products.

That markets are capable of efficiently allocating resources and stabilizing the price of an asset implies that the markets must, in some sense, take into account all information affecting the assets price. This is the essential content of the celebrated

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http://dx.doi.org/10.1016/j.physa.2015.03.032
0378-4371/

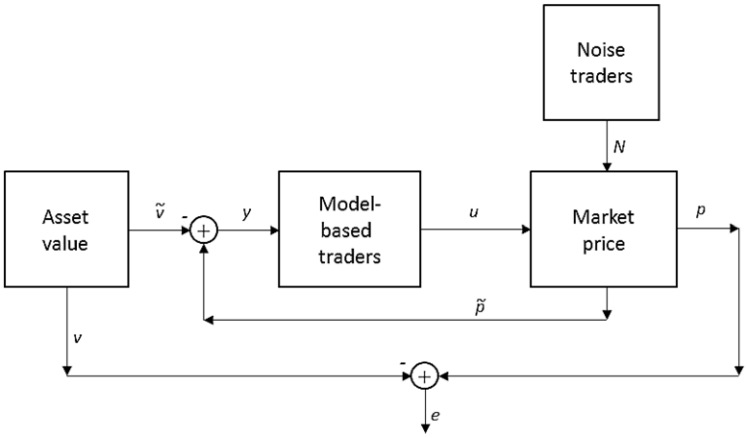

Fig. 1. The full value discovery process model for tâtonnement considered in the present article. Value discovery requires that $\lim_{t\to\infty} e(t) = 0$.

Efficient Market Hypothesis (EMH) due to Samuelson [3], Fama [4] and others. Although there exist many forms of the EMH, in broad terms they all assert that a market is efficient if prices immediately, for all practical purposes, reflect all relevant information about the assets on the market. The EMH thus requires that, on average, the population is always correct about the price (even if no single person is) and as new information appears, the market participants revise their expectations appropriately to maintain this state of affairs.

The degree to which the EMH holds true in practice has been debated in the academic literature over the course of decades (see Ref. [5] for a review). The observation that perfectly efficient asset prices imply purely random price fluctuations [3], and the subsequent conflicting rejection of the random walk property of observed asset prices [6], the existence of bubbles and crashes in asset prices [7], and the unusual profitability of simple technical strategies (see e.g. Ref. [8]), are among the key sources for criticism for the EMH. A conclusion of these studies is that the degree to which markets are efficient is likely not constant over time. In particular, as new information is being processed by market participants, there is a transient period (of unknown and varying duration) during which price may not reflect true value.

In spite of the progress made on understanding the nature of market efficiency, the actual mechanism by which prices adjust to new information – i.e. information processing by market participants during the tâtonnement process – appears to be relatively unknown [9]. In particular, to the author's knowledge there is no comprehensive mathematical model for price discovery based on the market participants' behavior. The purpose of this article is to fill this gap by presenting such a model for efficient markets. Our model explains, in rigorous terms, what it means for the markets to incorporate all available information about an asset, and provides necessary and sufficient conditions for value-based price discovery. The significance of such a model is not only in its ability to explain the behavior of market participants during tâtonnement but also, via the presented necessary conditions for EMH to hold, in the new directions it provides for testing – and perhaps rejecting – the hypothesis in practice.

1.2. Contribution of this article

The model developed in Section 2 allows us to formulate the entire Walrasian tâtonnement process as a robust output regulation problem. We can then invoke the celebrated Internal Model Principle of control theory (see e.g. Refs. [10,11]) to establish the main result of the article in Theorem 1. It shows that under No Bubble Conditions, the market for an asset $\mathcal{A}$ is efficient precisely when the following two conditions hold true:

1. The traders, as a group, can identify any mispricing in asset $\mathcal{A}$;
2. The traders, as a group, incorporate an internal model of the value process for $\mathcal{A}$.

A remarkable feature of our model is that, besides linear deterministicity discussed below, we make relatively few assumptions about the specific structure of the markets and about the arrangement of the individual traders and investors. As demonstrated in Fig. 1, we essentially treat the investors and the market as interconnected "black boxes" whose dynamical properties result from the interaction of – potentially a vast number of – individuals. Consequently, our modeling framework can simultaneously incorporate any number of traders with different trading strategies. These can include, among others, arbitrage strategies, value-driven ones whereby the traders' actions are driven by a perceived price–value discrepancy, and momentum-based ones, depending on positive feedback, without any regard to the specific design of their individual trading strategies. Further, it is remarkable that the occurrence of price–value discovery in our model does not depend on all market participants being rational. Indeed, part of the net demand–supply affecting the current price level in the presented model results explicitly from potentially irrational investors. In addition to this, our modeling approach to the market place dynamics allows us to incorporate a number of typical market microstructure models, such as the "law of the market" considered by Mosetti [12].