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Testing for detailed balance in a financial market

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HIGHLIGHTS

- We analyze a historical returns distribution of a financial price–time series.
- A test is devised to check for a property known as equilibrium in neo-classical economic theory.
- The analysis method is verified by applying it to a series with exact detailed balance and to a random one.
- We interpret the test results as revealing only a hint at equilibrium, but showing more resemblance to a random situation.

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ABSTRACT

We test a historical price–time series in a financial market (the NASDAQ 100 index) for a statistical property known as detailed balance. The presence of detailed balance would imply that the market can be modeled by a stochastic process based on a Markov chain, thus leading to equilibrium. In economic terms, a positive outcome of the test would support the efficient market hypothesis, a cornerstone of neo-classical economic theory. In contrast to the usage in prevalent economic theory the term equilibrium here is tied to the returns, rather than the price–time series. The test is based on an action functional S constructed from the elements of the detailed balance condition and the historical data set, and then analyzing S by means of simulated annealing. Checks are performed to verify the validity of the analysis method. We discuss the outcome of this analysis.

1. Introduction

Much of contemporary economic theory is dominated by the neo-classical paradigm of an efficient market. For definiteness, considering financial markets, this implies that the market participants (traders), have immediate and complete access to market information, like stock prices, sales volumes etc., and engage in rational behavior (trade decisions), aiming to maximize their self interest. It is argued that this situation will then lead to some kind of equilibrium state of the market, where the actual price of a financial instrument reflects its real market value at all times [1]. Hence, the efficient market hypothesis is seen as the cause for a perceived equilibrium in the market. In this context, equilibrium here means that the price fluctuates stochastically about some average value. The fluctuations, caused by the interactions of many traders, will influence the price only on a short time scale.

In an alternative scenario a market may be in an off-equilibrium state [2]. This paradigm would allow for dramatic price changes, catastrophic crashes in particular. A signature feature is a power law behavior of the returns distribution for extreme events, much resembling the Gutenberg–Richter law for earthquakes. This suggests that methods from the field of critical systems [3] might be fruitful. In the context of a financial time series, this situation can also emerge from a self-organized critical state [4,5]. Across the disciplines, ranging from neo-classical economists to econophysicists, the evidence

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for markets being off-equilibrium is not uniformly recognized. In this paper, we hope to make available a quantitative measure for the degree of non-stationarity that could provide answers to researchers of every couleur across the above spectrum. Also, the issue may not be as clear cut, some markets may be more off-equilibrium than others.

We are here interested in the question of whether a real market does reveal signs of equilibrium. Our approach is entirely based on the analysis of empirical, historical, data. For this purpose an operational definition of 'equilibrium' in a financial time series, the subject of our investigation, is required. We will motivate our choice (somewhat different from customary use) in Section 2, and then define the criterion for the test in Section 3. We discuss the numerical implementation and outcome of the test in Section 4, closing with Section 5, which contains the conclusion.

2. Motivation

For the sole purpose of motivating our criterion, we briefly reflect on the well-known Metropolis algorithm [6] which is a standard tool in numerical simulation for generating sets of random numbers according to a given probability density function. The algorithm is one way of producing a Markov chain of numbers [7,8], say

$$\cdots \leftarrow r(i+1) \leftarrow r(i) \leftarrow r(i-1) \leftarrow \cdots, \tag{1}$$

where a (real) value $r(i)$, at simulation 'time' counter i, is generated from the preceding one through a stochastic process. The latter involves a conditional probability density function, say $W(r' \leftarrow r)$. In the Metropolis algorithm it is constructed from a base probability distribution function, say $w(r)$, by creating a trial value r' and then accepting or rejecting it as the next value $r' \leftarrow r$ in the chain as determined by $W(r' \leftarrow r)$. The ensuing Markov chain will eventually, in the limit $\infty \leftarrow i$, produce values $r = r(i)$ distributed according to the base probability density function $w(r)$. Having converged to $w(r)$, the chain is said to have reached 'equilibrium'. A property known as detailed balance

$$W(r' \leftarrow r)w(r) = W(r \leftarrow r')w(r') \tag{2}$$

is a sufficient condition for the chain to reach equilibrium. The Metropolis algorithm makes a very specific choice for $W(r' \leftarrow r)$. However, we will make no use of it until later in Section 4 when we verify and discuss our results.

We here adopt the detailed balance condition (2) as the criterion to be tested for in a historical data set. Strictly speaking, it is 'only' a sufficient condition for equilibrium. On the other hand, the notion of equilibrium is rather fuzzy as used in an economic context. Detailed balance has the advantage of providing us with an operational, though possibly narrow, definition of equilibrium, which we will, nevertheless, use within this paper. As explained below, its numerical implementation will provide us with a rigorous analysis tool.

3. Testing historical data

The data set for our analysis is the price–time series of the NASDAQ 100 stock index. We use data from 2005-Aug-26 to 2008-Aug-25. The size of the set is $N = 266906$ [9]. Within normal trading hours (Monday through Friday, 09:30 h to 16:00 h) this translates to an average of about 1.14 min between trades. The price–time series is shown in Fig. 1. The counter i starts from $i = 0$ and is incremented by one every time a new quote emerges, $i = 0, 1 \ldots m = N - 1 = 266905$. A commonly used derived measure is the returns

$$r(i) = \log(p(i)/p(i-1)) \quad \text{for } i = 1 \ldots m, \tag{3}$$

where log here means the natural logarithm. The returns corresponding to Fig. 1, are displayed in Fig. 2. The original price–time series $p(i)$ can be easily reconstructed from $r(i)$ by way of recursion, $p(i) = p(i-1)\exp r(i)$, given the initial condition, which is $p(0) = 1565.87$ USD.

Obviously, as the price is strongly changing with time, see Fig. 1, the notion of equilibrium cannot apply directly to the price, at least not on the time scale considered. Adopting the naive notion that equilibrium manifests itself as a stable mean value subject to some fluctuations, the price–time series of Fig. 1 clearly rules out that condition. Thus, in contrast to mainstream economic practice, we will rather test the returns time series for equilibrium (viz. detailed balance). A glance at Fig. 2 shows that this is a much more reasonable starting point. In this sense, we deviate from the colloquial use of the term equilibrium in an economic context.

The extraction of the transition probability density $W(r' \leftarrow r)$ in (2) from the historical data set Fig. 2 is illustrated in Fig. 3. The pair of a return event $r(i)$ and its immediate successor $r(i+1)$ gives rise to a dot in one of the square bins of Fig. 3. The number of dot counts, in a particular square bin, then is an estimator for the transition probability density $W(r' \leftarrow r)$ in (2), up to a normalization factor. Within intervals $r \in [-0.02, +0.02]$, for each axis, we chose a discretization of $\Delta r = 0.0016$, which translates into $N = 25$ bins in each direction, or a total of 625 square bins. The data are very heavily peaked at the center, leading to saturation near the origin. The counts displayed in Fig. 3 accommodate all but 7 of the returns events of the data set. Fig. 4 is just a detail of Fig. 3 at a smaller scale.

It is convenient to adapt our notation to the discretization. Thus let $x, y \in \mathbb{N}$ denote discrete bin counters, $x, y = 1 \ldots N$, so each square bin is labeled by a pair (x, y). Thus $W(x, y)$ is the number of counts in square bin (x, y). Let $w(x)$ be the