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Optimal oil production and the world supply of oil

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ABSTRACT

We study the optimal oil extraction strategy and the value of an oil field using a multiple real option approach. The numerical method is flexible enough to solve a model with several state variables, to discuss the effect of risk aversion, and to take into account uncertainty in the size of reserves. Optimal extraction in the baseline model is found to be volatile. If the oil producer is risk averse, production is more stable, but spare capacity is much higher than what is typically observed. We show that decisions are very sensitive to expectations on the equilibrium oil price using a mean reverting model of the oil price where the equilibrium price is also a random variable. Oil production was cut during the 2008–2009 crisis, and we find that the cut in production was larger for OPEC members, for countries facing a lower discount rate, and for countries whose governments' finances are less dependent on oil revenues. However, the net present value of a country's oil reserves would be increased significantly (by 100%, in the most extreme case) if production was cut completely when prices fall below the country's threshold price. If several producers were to adopt such strategies, world oil prices would be higher but more stable.

1. Introduction

In this paper we investigate the optimal oil extraction strategy of a small oil producer facing uncertain oil prices. We use a multiple real option approach. Extracting a barrel of oil is similar to exercising a call option, *i.e.* oil production can be modeled as the right to produce a barrel of oil with the payoff of the strategy depending on uncertain oil prices. Production is optimal if the payoff of extracting oil exceeds the value of leaving oil under the ground for later extraction (the continuation value). For an oil producer, the optimal extraction path corresponds to the optimal strategy of an investor holding a multiple real option with finite number of exercises (finite reserves of oil). At any single point in time, the oil producer is also limited in the number of options he can exercise, because of capacity constraints.

Our first contribution is to present the solution to the stochastic optimization problem as an exercise rule for a multiple real option[1] and to solve the problem numerically using Monte Carlo methods developed by Longstaff and Schwartz

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[1] We use the term 'real option' following Dixit and Pindyck's definition that real options are "opportunities to acquire real assets" (1994, p. 7) — in our case, the opportunity to extract and own a barrel of oil.

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(2001), Rogers (2002), and extended by Aleksandrov and Hambly (2010), Bender (2011), and Gyurkó et al. (2011). The Monte Carlo regression method is flexible and it remains accurate even for high-dimensionality problems, *i.e.* when there are several state variables, for instance when the oil price process is driven by two state variables, when extraction costs are stochastic, or when the size of reserves is a random variable.

We solve the real option problem for two countries with similar capacity profiles but vastly different reserves (Brazil and U.A.E.) and compute the threshold below which it is optimal to defer production. In our baseline model, we find that Brazil, which has small reserves, should only produce when prices are high (higher than US$ 73 per barrel at 2000 constant prices), whereas for the U.A.E., full production is optimal as soon as prices exceed US$ 39. Optimal production is found to be volatile given the stochastic process of oil prices. As a result, we show that the net present value of oil reserves would be substantially higher if countries were willing to vary production when oil prices change. This result has important implications for oil production policy and for the design of macroeconomic policies that depend on inter-temporal and inter-generational equity considerations. Loosely extending our partial equilibrium analysis to general equilibrium, this result could imply that the world supply curve would be very elastic to prices were all countries to optimize production as in the baseline model—and thus, prices would tend to be higher but much less volatile.

We investigate why observed production is not as volatile as what is predicted by the baseline calibration of the model. One possible explanation is that producers are risk averse. Under this assumption, production is accelerated and is more stable, but a risk averse producer should also maintain large spare capacity, a result at odds with the evidence that oil producers almost always produce at full capacity. A second potential explanation is that producers are uncertain about the actual size of their oil reserves. Using panel data on recoverable reserves, we show however that, historically, this uncertainty has been diminishing with time and therefore this explanation is incomplete, since even mature oil exporters maintain low spare capacity. A third explanation may be that the oil price process, and in particular the equilibrium oil price, is unknown to the decision makers. Indeed, the optimal reaction to an increase in oil prices depends on whether the price increase is perceived to be temporary or to reflect a permanent shift in prices. If shocks are known to be primarily temporary, production should increase in the face of oil price increases. But if shocks are thought to be accompanied by movements in the equilibrium price, the continuation value jumps at the same time as the immediate payoff from extracting oil. In that case an increase in price may not result in an increase in production. Faced with uncertain views on the optimal strategy, the safe decision might well be to remain prudent with changes in production.

In practice, world oil production is partially cut in the face of negative demand shocks. The last section of the paper investigates whether the reduction in oil production during the 2008–2009 crisis can be explained by the determinants predicted by the model. We find that the cut in production was larger for OPEC members, for countries facing a lower discount rate, as predicted by the model, and for countries whose governments' finances are less dependent on oil revenues.

Section 2 provides a survey of the related literature on optimal production and real options, while Sections 3–5 cover the model formulation and calibration. Section 6 describes briefly the oil sector in the two countries used as applications. Sections 7 presents the basic set of results and Section 8 discusses some limitations of the model. Section 9 investigates the determinants of production strategies during the 2008–2009 crisis and Section 10 concludes on the price-elasticity of the world supply of oil.

2. Related literature

2.1. Optimal oil production

The study of the economy of non-renewable resource extraction started with Hotelling (1931), who showed in a deterministic general equilibrium model that the price of the resource would grow at the rate of interest in competitive markets with constant extraction costs. General equilibrium models later included the effect of uncertainty in technology, the size of the resources, or the availability of substitutes. Partial equilibrium models in which the prices are given, but the decision to extract is a function of the stochastic price process, have a shorter history in the non-renewable resource literature, starting with Tourinho (1979a). Tourinho (1979a, 1979b), analyzed for the first time the valuation of resources in the context of a real 'call' option to exploit a field, using the Black and Scholes framework.

Paddock et al. (1988) later developed a model that became a popular approach for decisions on upstream oil investments, in which a company has the option to explore an area and in case oil is discovered, to commit to an immediate development investment before a given date (the time to expiration). If the firm does not exercise the option to develop the field until this date, the firm must return the concession rights back to a national authority.[2] The model took into consideration resource depletion when estimating the value of the oil field. However, the issue of when to extract the resource after the field is developed is completely absent since the only decision to take is the optimal timing to develop the field.

Cherian et al. (2000) studied optimal production of a nonrenewable resource as a control problem in continuous time. The authors solved the Bellman nonlinear partial differential equation (PDE) numerically using the Markov chain approximation technique of Kushner (1977) and Kushner and Dupuis (1992). The cost of extraction in Cherian et al. (2000) is a function of

[2] Real option models are surveyed in Dixit and Pindyck (1994) and, for applications to oil investments, in Dias (2004).