Insurance: Mathematics and Economics 64 (2015) 396–408

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Equilibrium investment strategy for defined-contribution pension schemes with generalized mean–variance criterion and mortality risk

Huiling Wu [a], Yan Zeng [b,*]

[a] China Institute for Actuarial Science, Central University of Finance and Economics, Beijing 100081, PR China

[b] Lingnan (University) College, Sun Yat-sen University, Guangzhou 510275, PR China

HIGHLIGHTS

- The multi-period equilibrium investment strategy for DC pension schemes is studied.
- A generalized mean–variance criterion is adopted in the objective function.
- Stochastic salary flow and stochastic mortality rate are considered in the model.
- Analytical expressions for equilibrium strategy and value function are derived.
- The effects of the mortality risk on our derived results are illustrated.

ARTICLE INFO

Article history:
Received January 2015
Received in revised form
June 2015
Accepted 24 July 2015
Available online 31 July 2015

Keywords:
Defined-contribution pension scheme
Equilibrium investment strategy
Mortality risk
Generalized mean–variance criterion
Time-inconsistency

ABSTRACT

This paper studies a generalized multi-period mean–variance portfolio selection problem within the game theoretic framework for a defined-contribution pension scheme member. The member is assumed to have a stochastic salary flow and a stochastic mortality rate, and is allowed to invest in a financial market with one risk-free asset and one risky asset. The explicit expressions for the equilibrium investment strategy and equilibrium value function are obtained by backward induction. In addition, some sensitivity analysis and numerical illustrations are provided to show the effects of mortality risk on our results.

1. Introduction

According to the contract of contributions and benefits, enterprise pension schemes are mainly divided into defined-benefit (DB) schemes and defined-contribution (DC) schemes. In a DB scheme, financial risk and longevity risk are mainly faced by the sponsor. In this case, the benefits are predetermined while the contributions are initially set and then subsequently adjusted to keep the scheme in balance. However, in a DC scheme, the members mainly face financial and longevity risks, with the contributions fixed in advance and the benefits dependent on the investment performance of the fund during the accumulation phase. Nowadays, many countries are making efforts to transfer longevity risk and are finding ways to guarantee the sustainability of a sufficient retirement income. According to the contract, DC plans have an advantage over DB plans in that they can ease the pressure on social security programs by transferring investment and longevity risks to the retirees. As a result, DC schemes are playing an increasingly important role in social security programs, with many countries preferring DC schemes over DB schemes.

For a DC scheme member, the main income after retirement comes from the pension fund, and is therefore heavily dependent on the investment performance of the fund during the accumulation phase. Hence, the analysis of the asset allocation in the accumulation phase of a DC scheme is very important. Numerous studies have examined the optimal investment strategies in the accumulation phase of a DC scheme using different objective functions and financial market settings. For example, to maximize the expected utility from the terminal wealth under the CRRA or CARA utility functions, Cairns et al. (2006) investigate the asset allocation strategies to DC scheme members with asset, salary and interest rate risk, and propose a novel form of terminal utility function

* Corresponding author. Tel.: +86 20 84110516; fax: +86 20 84114832.
E-mail addresses: sunnyling168@hotmail.com (H. Wu), zengy36@mail.sysu.edu.cn (Y. Zeng).

http://dx.doi.org/10.1016/j.insmatheco.2015.07.007
0167-6687/

by incorporating habit formation. Zhang et al. (2007) and Zhang and Ewald (2010) aim to maximize the power utility of the terminal value of a DC pension fund under inflation risk. Giacinto et al. (2011) propose and investigate a model of optimal allocation for a DC pension plan with a minimum guarantee in the continuous-time setting. Korn et al. (2011) study an optimal portfolio selection problem for a DC scheme member in a hidden Markov-modulated economy. Within the framework of prospect theory, Blake et al. (2013) consider the optimal dynamic investment strategies for DC pension plans when the plan members are loss averse.

Recently, some scholars have also examined the optimal investment strategy for a DC pension plan with the mean–variance criterion proposed by Markowitz (1952). Højgaad and Vigna (2007) compare a mean–variance model with a target-based model, and show that the target-based model can be formulated as a mean–variance model. Nkeki (2013) studies a mean–variance DC pension management problem with time-dependent salary, and compares the optimal portfolios under quadratic utility function, power utility function and exponential utility function. He and Liang (2013) introduce the return of premium clauses into the portfolio model with the mean–variance criterion for a DC pension plan during the accumulation phase, and derive a time-consistent investment strategy within the game theoretic framework. Menoncin and Vigna (2013) consider a mean–variance investment problem for a DC pension plan with a stochastic interest rate in the accumulation phase. Guan and Liang (2015) generalize this problem to the case with stochastic interest rate, stochastic volatility and stochastic salary. Vigna (2014) compares the mean–variance efficient portfolios with the optimal portfolios maximizing the expected CARA and CRRA utilities, which are proved to be not mean–variance efficient. Yao et al. (2014) consider a multi-period mean–variance investment problem for the accumulation phase of a DC pension scheme. For more information about optimal portfolio selection for a DC pension scheme under the mean–variance criterion, interested readers are referred to Vigna (2009) and Nkeki (2012).

However, in the above-mentioned literature, except for He and Liang (2013), the optimal investment strategy with the mean–variance criterion is time-inconsistent, which is only optimal at the initial time. That is, the optimal strategy at time m does not agree with that at time n, where $n > m$, because the mean–variance criterion does not have the iterated expected property. Therefore, the optimal strategy in the classical mean–variance model is usually called the pre-commitment strategy. In recent years, the pre-commitment strategy has been criticized for lacking rationality on the basis that investment psychology and taste change over time. For this reason, there has seen a recent upsurge of interest in studying the time-consistent strategy for the mean–variance problem, see Stroz (1956), Björk and Murgoci (2010), Basak and Chabakauri (2010), Wu (2013), Björk et al. (2014), Björk and Murgoci (2014), Bensoussan et al. (2014) and references therein.

To the best of our knowledge, for the discrete-time multi-period mean–variance investment problem in the accumulation phase of a DC pension scheme, only Yao et al. (2014) consider an optimal pre-commitment investment strategy that is time-inconsistent, and no studies have examined the corresponding time-consistent investment strategy. Therefore, this paper presents the first study of this strategy. Specifically, we consider a multi-period mean–variance investment problem for a DC scheme member in the accumulation phase, and attempt to derive the time-consistent equilibrium investment strategy within the game theoretic framework. We assume that the member has a stochastic salary flow and a stochastic mortality rate, and she can invest her wealth in a financial market consisting of one risk-free asset and one risky asset. Due to the mortality risk, the time horizon of the member is uncertain. In addition, we assume that the objective of the scheme member is to maximize the weighted sum of a linear combination of the expectation and variance of the terminal wealth. In our paper, the objective function of the member varies over time, according to the weighted coefficients measured by the corresponding exit probabilities. Similar to Björk and Murgoci (2010, 2014) and Bensoussan et al. (2014), we take this problem as a non-cooperative game and derive the closed-form expressions for the equilibrium strategy and equilibrium value function by backward induction. We also present some special cases of our results and the relationship between the expectation and variance of the terminal wealth. Moreover, we provide some sensitivity analysis and numerical illustrations, which show that some properties of the optimal strategy in the case without mortality risk do not hold in the case with mortality risk.

Compared with Yao et al. (2014) and other existing literature, this paper makes four main contributions: (1) a generalized mean–variance criterion is first introduced into the optimal investment model of DC pension schemes, which makes our optimization problem more general; (2) the mechanism of defining the exit probabilities is quite different from that in the existing literature, in that we assume that the exit probabilities depend on the starting times and the future times, which also makes our optimization problem time-inconsistent; (3) the time-consistent equilibrium investment strategy is first considered and derived explicitly for the multi-period mean–variance investment problem in the accumulation phase of a DC pension scheme; and (4) to the best of our knowledge, we are the first to consider the time-consistent equilibrium strategy for the multi-period mean–variance investment problem with uncertain time horizon.

The remainder of this paper is organized as follows. The assumptions and problem formulation are described in Section 2. The equilibrium investment strategy and equilibrium value function are derived explicitly by backward induction in Section 3, and some special cases of our results and properties of our equilibrium strategy are also presented. The expectation and variance of the terminal wealth and the relationship between them are provided in Section 4. In Section 5, we provide some numerical illustrations to show the effects of mortality risk on our results and produce some interesting findings. Conclusions are given in Section 6. Proofs of the propositions and theorems are given in Appendices A–E.

2. Problem formulation

Consider a financial market that consists of one risk-free asset and one risky asset. Over period $[n, n+1)$, the risk-free asset has a deterministic and positive return r_n^f and the risky asset has a random return R_n. The member enters the financial market at time 0 and plans to invest her wealth in the market within T consecutive time periods. However, due to the mortality risk, the member does not know exactly the time when she will exit the market. The dynamics of the member's salary before her death are given as:

$$Y_{n+1} = Q_n Y_n, \quad n = 0, 1, \dots, T-1, \tag{2.1}$$

where Y_n is the member's salary at time n and Q_n is an exogenous nonnegative random variable representing the stochastic growth rate of the member's salary over period $[n, n+1)$. At time n, contributions are paid as a nonnegative deterministic proportion c_n of Y_n. Denote $\eta_n = \mathrm{E}(Q_n)$, and suppose that Q_n is independent of Q_m, $\forall m \neq n$. Let $\pi := \{\pi_n, n = 0, 1, \dots, T-1\}$ be an investment strategy, where π_n is the amount invested in the risky asset at time n. Then the wealth of the member under strategy π evolves over time according to:

$$\begin{aligned} W_{n+1}^{\pi} &= \left(W_n^{\pi} + c_n Y_n - \pi_n\right) r_n^f + R_n \pi_n \\ &= W_n^{\pi} r_n^f + \pi_n R_n^e + c_n r_n^f Y_n, \quad n = 0, 1, \dots, T-1, \end{aligned} \tag{2.2}$$