Insurance: Mathematics and Economics 55 (2014) 105–115

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Optimal reinsurance and investment strategies for insurer under interest rate and inflation risks

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HIGHLIGHTS

- Establish risk model with stochastic interest rate, inflation index, bonds and TIPS.
- Study the optimal reinsurance and investment problem under maximizing CRRA utility.
- Derive closed-forms of the optimal utility, reinsurance and investment strategies.
- Give a sensitivity analysis to clarify the behavior of the risk model.

ARTICLE INFO

Article history:
Received October 2013
Received in revised form
January 2014
Accepted 8 January 2014

JEL classification:
G11
C61
G32

MSC:
91B30
91B70
91B16
91G10
93E20

Submission Classifications:
IE13
IE12
IM52
IB91
IE53
IE43

Keywords:
Optimal proportional reinsurance strategy
Optimal investment strategy
CRRA utility
Stochastic dynamic programming
Stochastic inflation index
Stochastic interest rate

ABSTRACT

In this paper, we investigate an optimal reinsurance and investment problem for an insurer whose surplus process is approximated by a drifted Brownian motion. Proportional reinsurance is to hedge the risk of insurance. Interest rate risk and inflation risk are considered. We suppose that the instantaneous nominal interest rate follows an Ornstein–Uhlenbeck process, and the inflation index is given by a generalized Fisher equation. To make the market complete, zero-coupon bonds and Treasury Inflation Protected Securities (TIPS) are included in the market. The financial market consists of cash, zero-coupon bond, TIPS and stock. We employ the stochastic dynamic programming to derive the closed-forms of the optimal reinsurance and investment strategies as well as the optimal utility function under the constant relative risk aversion (CRRA) utility maximization. Sensitivity analysis is given to show the economic behavior of the optimal strategies and optimal utility.

1. Introduction

Optimal investment strategy for insurer has recently become an important subject. The insurer can participate in the financial market to avoid risk. More recently, many literatures have studied maximizing the utility of terminal value or minimizing the

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http://dx.doi.org/10.1016/j.insmatheco.2014.01.007

probability of ruin for the insurer. Browne (cf. Browne, 1995) initiated the study of explicit solution for a firm to maximize the exponential utility of terminal wealth and minimize the probability of ruin with its surplus process given by the Lundberg risk model. For different claim sizes of insurers, the optimal strategy was given by the Bellman equation in Hipp and Plum (2000) to minimize the ruin probability. Wang, Xia and Zhang (cf. Wang et al., 2007) efficiently applied martingale method to study the optimal portfolio selection for insurer under the mean–variance criterion as well as the expected constant absolute risk aversion (CARA) utility maximization. The readers are referred to, for example, Yang and Zhang (2005), Wang (2007), Liu and Yang (2004), Bai and Guo (2008) and references therein.

In addition to the risk of market, the insurer also takes into account the risk of insurance. The risk of insurance cannot be avoided by singly investing in the bond and other assets in the market. However, the business of reinsurance provides a way for the insurer to hedge this risk, and this way has also recently drawn much concern. The business of reinsurance comes up in different forms. Quota-share reinsurance and investment were originally considered by Promislow and Young (cf. Promislow and Young, 2005). Proportional reinsurance was accessible in Bäuerle (2005) in which the author minimized the expected quadratic distance of the terminal value over a positive constant and successfully solved the related mean–variance problem. Zeng and Li (cf. Zeng and Li, 2011) essentially got the mean–variance efficient frontier of the diffusion model with multiple risky assets in the case of proportional reinsurance. The stock price in the above models generally follows a geometric Brownian motion and the market price of the risk correlated with the stock is constant. But in the real market, the stock price may have other features, for example, stochastic volatility. Liang, Yuen and Guo (cf. Liang et al., 2011) characterized the instantaneous rate of the stock by Ornstein–Uhlenbeck process and derived the optimal reinsurance and investment strategies. The constant elasticity of variance (CEV) model was established in Gu et al. (2012) in which the insurer can buy excess-of-loss reinsurance. In Bäuerle and Blatter (2011), both the surplus of the insurer and the stock index in the market followed the Lévy process, and optimal investment and reinsurance policies were explicitly derived. Moreover, the optimal investment strategy was beautifully solved by Badaoui and Fernández (cf. Badaoui and Fernández, 2013) when the instantaneous rate and the volatility were related with a common stochastic factor.

Based on the investment and reinsurance strategy, the insurer can successfully avoid its risk. However, the time of investment may be long for the insurer, so it is natural to take the risk of interest rate into account. So far, few literature is available for insurer under stochastic interest rate. Elliott and Siu (cf. Elliott and Siu, 2011) used the so called game theoretic approach to find the best allocations in the market when the interest rate was given by a regime-switching model. In fact, most of the work of investment under stochastic interest focus on portfolio selection. In the case of stochastic interest rate, zero coupon bonds, delivering a fixed return of \$1 at maturity, are issued in the market to hedge the risk of interest rate. With the help of zero coupon bonds, we can establish a complete market. Bajeux-Besnainou and Portait (cf. Bajeux-Besnainou and Portait, 1998) first solved the portfolio selection problem when the instantaneous interest rate was stochastic. They introduced the pricing kernel and derived the mean–variance efficient frontier under the generalized Vasicek model. Bajeux-Besnainou, Jordan and Portait (cf. Bajeux-Besnainou et al., 2003) considered a case when the interest rate followed an Ornstein–Uhlenbeck process and got the optimal investment strategies to maximize CRRA and hyperbolic absolute risk aversion (HARA) utility for investors by martingale methods. Mean–variance problem with extended Cox–Ingersoll–Ross (CIR) stochastic interest rate model was studied by Ferland and Waiter (cf. Ferland and Waiter, 2010). Besides, Boulier, Huang and Taillard (cf. Boulier et al., 2001), Josa-Fombellida and Rincón-Zapatero (cf. Josa-Fombellida and Rincón-Zapatero, 2010) solved the optimal investment problem under stochastic interest rate in defined contribution (DC) and defined benefit (DB) pension plans, respectively.

Also, the inflation risk is an important factor in the long run of investment. To hedge the inflation risk, in the case of optimal asset allocation with inflation, Treasury Inflation Protected Securities (TIPS) are needed. There are many TIPS in practice, in which people often use inflation-indexed zero coupon bond in the market. The model of inflation often includes nominal interest rate, real interest rate and the inflation index. The inflation index is also a factor to characterize the connection between the nominal market and the real market. The most famous equation between them is given by the famous Fisher equation. Jarrow and Yildirim (cf. Jarrow and Yildirim, 2003) made a breakthrough in establishing the Jarrow–Yildirim (JY) model to characterize the inflation index, the forward nominal interest rate and forward real interest rate. Brennan and Xia (cf. Brennan and Xia, 2002) modeled the inflation index in a different framework and obtained the optimal investment strategies under inflation. Besides, Zhang, Korn and Ewald (cf. Zhang et al., 2007) extended the Fisher equation under the risk-neutral measure and used the martingale method to derive the optimal allocations. Later, Han and Hung (cf. Han and Hung, 2012) first introduced the risks of inflation and interest rate in a DC pension fund model.

Unfortunately, as far as we are concerned, no literature of insurer cares about the above two important risks of market at the same time. But when we concern the optimal reinsurance and investment strategies for a long time, the both risks of interest rate and inflation should be included. More precisely, in this paper, we will concentrate on studying the optimal reinsurance and investment problem for an insurer under risks of interest rate and inflation. The objective of the insurer is to maximize the expected CRRA utility of the terminal real wealth, where we assume that the nominal interest rate follows an Ornstein–Uhlenbeck process, the connections among real interest rate, nominal interest rate and the inflation index are given by the famous Fisher equation. To make the market complete and hedge the risk of market, zero-coupon bonds, TIPS and stocks are also included in the financial market. Moreover, we also assume that the proportional reinsurance is allowed. By using the stochastic dynamic programming method, we first derive the Hamilton–Jacobi–Bellman (HJB) equations for the problem, and then solve it by employing a variable change technique, finally get the closed-forms of the optimal reinsurance and investment strategies in the dynamic optimization problem. However, since the existence of insurance, we will not get a self-financing wealth process and this makes the problem very difficult. To handle this situation, auxiliary process will be introduced to make the market also self-financing, and the auxiliary process will help to solve the optimal reinsurance and investment problem for insurers.

The paper is organized as follows. The model of proportional reinsurance with stochastic nominal interest rate and inflation index is presented in Section 2, and the dynamics of zero coupon bonds and TIPS are also given. Section 3 introduces an auxiliary problem and derives the optimal reinsurance and investment strategies by stochastic dynamic programming. Section 4 provides a sensitivity analysis to clarify the behavior of our model. Section 5 is a conclusion.

2. The risk model

In this section, we obtain a financial market for an insurer with risks of inflation and interest rate. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbf{P})$ is a