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Time-consistent reinsurance–investment strategy for a mean–variance insurer under stochastic interest rate model and inflation risk

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HIGHLIGHTS

- Insurer's investment under stochastic interest rate and inflation index is studied.
- We consider reinsurance–investment problem under mean–variance criterion.
- We obtain time-consistent strategy and value function explicitly.
- The verification theorem is provided and proved.
- We compare the time-consistent strategy with the precommitment strategy.

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ABSTRACT

In this paper, we consider the time-consistent reinsurance–investment strategy under the mean–variance criterion for an insurer whose surplus process is described by a Brownian motion with drift. The insurer can transfer part of the risk to a reinsurer via proportional reinsurance or acquire new business. Moreover, stochastic interest rate and inflation risks are taken into account. To reduce the two kinds of risks, not only a risk-free asset and a risky asset, but also a zero-coupon bond and Treasury Inflation Protected Securities (TIPS) are available to invest in for the insurer. Applying stochastic control theory, we provide and prove a verification theorem and establish the corresponding extended Hamilton–Jacobi–Bellman (HJB) equation. By solving the extended HJB equation, we derive the time-consistent reinsurance–investment strategy as well as the corresponding value function for the mean–variance problem, explicitly. Furthermore, we formulate a precommitment mean–variance problem and obtain the corresponding time-inconsistent strategy to compare with the time-consistent strategy. Finally, numerical simulations are presented to illustrate the effects of model parameters on the time-consistent strategy.

1. Introduction

Reinsurance and investment are two important issues for an insurance company. Reinsurance can protect insurers against potentially large losses, while investment enables insurers to achieve his/her management objectives. Therefore, many optimization problems about reinsurance and investment with various objectives have risen in recent years. For example, Hipp and Plum (2000), Schmidli (2002) and Promislow and Young (2005) investigated the optimal reinsurance and investment problem for an insurer in the sense of minimizing the ruin probability. For the objective of expected utility maximization, Cao and Wan (2009) studied the optimal proportional reinsurance and investment problem of maximizing the expected exponential and power utilities from terminal wealth. Lin and Yang (2011) considered an insurer whose surplus process was governed by a jump–diffusion risk process and obtained the optimal reinsurance–investment strategy to maximize the expected exponential utility from terminal wealth. Liang and Bayraktar (2014) discussed the optimal reinsurance and investment problem in an unobservable Markov-modulated compound Poisson risk model. Besides, mean–variance criterion becomes another popular objective in the literature of optimal reinsurance and investment problems, see Bäuerle (2005), Delong and Gerrard (2007) and Bai and Zhang (2008). Traditional

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mean–variance optimization problem is a time-inconsistent problem where an optimal solution obtained at a time is no longer optimal as time goes forward into a future point, and the Bellman's principle of optimality does not hold. Since time-consistency is important for a rational decision maker, more and more researches develop the time-consistent strategy for the mean–variance problem. The main approach is to formulate the problem in a game theoretic framework, i.e., sitting at time t, the optimal strategy derived at time t agrees with that derived at time $t + \Delta t$. For more details, we can refer to Zeng and Li (2011), Zeng et al. (2013), Li and Li (2013) and references therein.

Later, some researchers introduced stochastic volatility models into the optimal reinsurance and investment problem. For instance, Gu et al. (2010), Lin and Li (2011), Liang et al. (2012) and Gu et al. (2012) applied the constant elasticity of variance (CEV) model to study the optimal proportional/excess-of-loss reinsurance and investment problem for an insurer with diffusion/jump–diffusion risk process, respectively. Li et al. (2012) began to use the Heston model to investigate the optimal proportional reinsurance and investment problem under the mean–variance criterion. Zhao et al. (2013) considered the optimal excess-of-loss reinsurance and investment problem for an insurer with jump–diffusion risk process under the Heston model. Zhu et al. (2015) discussed the optimal reinsurance and investment problem for an insurer in a defaultable market under the Heston model. Shen and Zeng (2015) studied the optimal reinsurance and investment problem for mean–variance insurer with square-root factor process.

However, most of the above-mentioned papers assume a constant level of the interest rate and ignore the inflation risk, which is contrary to practice according to the empirical results. Since the investment decision for an insurer may involve quite a long period, it is reasonable to take the risk of the interest rate into account. Recently, there are some researches about the portfolio selection and annuity contract under stochastic interest rate model. Korn and Kraft (2001) investigated the classical Merton problem with stochastic interest rate and derived the optimal investment strategy to maximize the constant relative risk aversion (CRRA) utility from terminal wealth. Li and Wu (2009) considered the CRRA utility maximization problem under stochastic interest rate and stochastic volatility model. Munk and Sørensen (2010) studied the dynamic asset allocation problem with stochastic interest rate and income. Besides, Josa-Fombellida and Rincón-Zapatero (2010) and Guan and Liang (2015) obtained the optimal investment strategy under stochastic interest rate model for the defined benefit (DB) and the defined contribution (DC) pension plan, respectively. So far, only few researches studied the optimal investment problem for an insurer with stochastic interest rate. For example, Li and Wu (2012) focused on the upper bound for the ruin probability and derived the optimal investment strategy by a pure probabilistic method with stochastic interest rate for an insurer. Guan and Liang (2014) discussed the optimal reinsurance and investment problem with the CRRA utility maximization under stochastic interest rate model.

In addition, inflation is also a common phenomenon in the long-term investment. For the research of the inflation risk, the model often includes nominal interest rate, real interest rate and the inflation index. The inflation index is a factor to characterize the connection between the nominal market and the real market. The asset allocation problem incorporating the inflation risk for the investor has been surveyed in some papers, see, Brennan and Xia (2002), Jarrow and Yildirim (2003), Munk et al. (2004) and Zhang et al. (2007) among others. Particularly, Han and Hung (2012) focused on the optimal investment problem for the DC pension plan with both interest rate and inflation risks. Later, Guan and Liang (2014) introduced the two kinds of risks into the optimal reinsurance and investment problem for an insurer.

In this paper, we extend the work of Guan and Liang (2014). Different from Guan and Liang (2014), we consider the optimal reinsurance and investment problem under the mean–variance criterion and derive the time-consistent strategy. In our model, the surplus process of the insurer is described by a Brownian motion with drift, and the insurer can transfer part of the risk to a reinsurer via proportional reinsurance or acquire new business. Moreover, stochastic interest rate and inflation risks are taken into account. The instantaneous nominal interest rate follows an Ornstein–Uhlenbeck process, and the inflation index is given by a diffusion process, which is partially correlated with the interest rate. To reduce interest rate and inflation risks, we add zero-coupon bonds and Treasury Inflation Protected Securities (TIPS) to the financial market. Besides the above two assets, the insurer can also invest in a risk-free asset and a risky asset. Since the existence of insurance business, the wealth process of the insurer is not a self-financing process. To deal with this situation, an auxiliary process is introduced to solve the optimal reinsurance and investment problem. Applying stochastic control theory, we provide and prove a verification theorem and establish the corresponding extended Hamilton–Jacobi–Bellman (HJB) equation. By solving the extended HJB equation, we derive the explicit expressions of the time-consistent reinsurance–investment strategy as well as the corresponding value function. To compare with the time-consistent strategy, we formulate a precommitment mean–variance problem and obtain the corresponding time-inconsistent strategy. Finally, numerical simulations are presented to illustrate the effects of model parameters on the time-consistent strategy.

The remainder of this paper is organized as follows. In Section 2, model formulation under the mean–variance criterion with stochastic interest rate and inflation risks is presented. In Section 3, we consider the time-consistent strategy of the problem. An auxiliary problem and the corresponding verification theorem are provided and proved. By solving an extended HJB equation, we derive the time-consistent reinsurance–investment strategy. Furthermore, we discuss the precommitment strategy for the mean–variance problem and compare the results under the two cases. In Section 4, sensitivity analysis and numerical simulations are presented to illustrate our results. Section 5 concludes this paper.

2. Model formulation

In this paper, we consider an insurer whose surplus process is described by a diffusion model. To understand the diffusion model better, we start with the classical Cramér–Lundberg (C–L) model, i.e.,

$$\mathrm{d}X(t) = c\mathrm{d}t - \mathrm{d}\sum_{i=1}^{N(t)} Y_i,$$

where c is the premium rate; $\{N(t), 0 \le t \le T\}$ is a homogeneous Poisson process with intensity $\lambda > 0$ and $N(t)$ represents the number of claims which have arrived up to time t; Y_i, $i = 1, 2, 3, \ldots$ represents the size of the ith claim and the claim sizes are assumed to be independent and identically distributed positive random variables and independent of $N(t)$. Denote by the first-order moment $\mathrm{E}[Y_i] = \mu_1$ and second-order moment $\mathrm{E}[Y_i^2] = \mu_2$. Assume that the premium rate is calculated according to the expected value principle, i.e., $c = \lambda\mu_1(1 + \eta)$, where $\eta > 0$ is the safety loading of the insurer.

In addition, the insurer is allowed to purchase proportional reinsurance or acquire new business (for example, acting as a reinsurer of other insurers, see Bäuerle, 2005) at each moment in order to control insurance business risk. The proportional reinsurance/new business level is associated with the value of risk