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A study of developing an input-oriented ratio-based comparative efficiency model

Ching-Kuo Wei [a], Liang-Chih Chen [b], Rong-Kwei Li [b], Chih-Hung Tsai [c,*]

[a] Department of Health Care Administration, Oriental Institute of Technology, 58, Sec. 2, Sihchuan Road, Pan-Chiao City, Taipei County 22061, Taiwan
[b] Department of Industrial Engineering and Management, National Chiao-Tung University, Hsinchu, Taiwan
[c] Department of Information Management, Yuanpei University, No. 306, Yuanpei Street, Hsin-Chu, Taiwan

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ABSTRACT

Data envelopment analysis (DEA) is a representative method to estimate efficient frontiers and derive efficiency. However, in a situation with weight restrictions on individual input–output pairs, its suitability has been questioned. Therefore, the main purpose of this paper is to develop a mathematical method, which we call the input-oriented ratio-based comparative efficiency model, DEA-R-I, to derive the input-target improvement strategy in situations with weight restrictions. Also, we prove that the efficiency score of DEA-R-I is greater than that of CCR-I, which is the first and most popular model of DEA, in input-oriented situations without weight restrictions to claim the DEA-R-I can replace the CCR model in these situations. We also show an example to illustrate the necessity of developing the new model. In a nutshell, we developed DEA-R-I to replace CCR-I in all input-oriented situations because it sets a more accurate weight restriction and yields a more achievable strategy.

1. Introduction

Data envelopment analysis (DEA) is one popular method for identifying efficient frontiers and evaluating efficiency. An efficient frontier is based on the concept of a non-dominated condition, which was first expressed by the Italian economist Pareto in 1927. This concept was adapted to production by Koopmans in 1951 and to evaluate efficiency by Farrell in 1957 (Cooper et al., 2002). Charnes, Cooper, and Rhodes (1978) applied linear programming (LP) to identify efficient frontiers and measure productivity. This method, which measures productivity by LP, is called "data envelopment analysis". They derived both an output-oriented (CCR-O) model and an input-oriented (CCR-I) model, which are not only the first but also most popular models of DEA. Many scholars have used DEA as the representative method to estimate an efficient frontier and measure productivity (Amirteimoori, 2007; Jahanshahloo, Hosseinzadeh Lotfi, & Zohrehbandian, 2005a). Over the past two decades, DEA has been established as a robust and valuable methodology (Chen & Ali, 2002; Liu & Chuang, 2009).

One advantage of DEA is objective weight selection, and there are many studies that focus on weight (Bernroider & Stix, 2007; Jahanshahloo, Soleimani-damaneh, & Nasrabadi, 2004; Jahanshahloo, Memariani, Hosseinzadeh, & Shoja, 2005b; Lotfi, Jahanshahloo, & Esmaeili, 2007; Wang, Parkan, & Luo, 2008). However, when applying the typical DEA model, which is based on $(\sum vx)/(\sum uy)$ or $(\sum uy)/(\sum vx)$, to a situation with weight restrictions on individual input–output pairs, its suitability is questionable. We take a case in hospitals as the example of the necessity of weight restrictions. Sickbeds, physicians, outpatients, inpatients, and surgery are important variables for hospital performance evaluations, where the sickbed variable contributes only to the inpatient and surgery variables but not the outpatient variable. In this situation, it is hard to assign a suitable weight restriction to an outpatient-sickbed pair. Golany and Roll (1989) argue that input–output pairs must correspond to an isotonicity assumption to avoid this problem. However, an isotonicity assumption represents a statistical rather than a causal relationship. For example, the statistical relationship between outpatient services and sickbeds is high, but the causal relationship between them is low. Therefore, conformance to the isotonicity assumption does not always avoid this problem. Dyson et al. (2001) argue that handling weight restrictions is still a pitfall in DEA applications from a theoretical perspective. Despic, Despic and Paradi (2007) claim that this kind of problem is difficult to solve with a typical DEA model and therefore developed DEA-R, a model to solve the problem of weight restriction. DEA-R is expressed as follows:

$$\hat{\hat{e}}_k = \max_{c_{(j,i)} \geqslant 0} \left\{ \sum_{j=1}^{s} \sum_{i=1}^{m} c_{(j,i)} r_{(j,i)k} \,\middle|\, \sum_{j=1}^{s} \sum_{i=1}^{m} c_{(j,i)} r_{(j,i)p} \leqslant 1, \right.$$
$$\left. \text{for } p = 1, 2, \ldots, n \right\}. \quad (1)$$

However, the DEA-R model developed by Despic et al. (2007) is an output-oriented model (we call it DEA-R-O). In some situations, we

* Corresponding author. Tel.: +886 3 6102338; fax: +886 3 6102343.
E-mail addresses: ietch@thit.edu.tw, imtch@mail.ypu.edu.tw (C.-H. Tsai).

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need an input-oriented model to provide an input-target improvement strategy with weight restrictions. Using Taiwan's private hospitals as an example again, the output was bounded by National Health Insurance; they have to adopt an input-targeted improvement strategy (reduce inputs), rather than an output-targeted strategy, to improve their efficiency. Therefore, a new mathematical method of deriving the input strategy (we call it input-oriented DEA-R, or DEA-R-I) has been developed.

In addition, the DEA-R-I seems to substitute for CCR-I in input-oriented situations without weight restrictions because the efficiency score of DEA-R-I is greater than or equal to than CCR-I when the relationship between DEA-R-O and CCR-O is unclear. According to Despic et al. (2007), the efficiency score of DEA-R-O with no weight restrictions is sometimes higher and sometimes lower than the efficiency score of CCR-O. This drawback prevents DEA-R-O from replacing CCR-O in a situation without weight restrictions. But, based on our study, we found two factors that cause this efficiency score discrepancy. The first is a more flexible selection of optimum weight, which affects the efficiency score of DEA-R-O higher than the efficiency score of CCR-O, while the second is the sum of the output-oriented ratio $\sum(w \times \frac{y}{x})$, which affects the efficiency score of DEA-R-O less than the efficiency score of CCR-O. Since we will use the sum of the input-oriented ratio $\sum\left(w \times \frac{x}{y}\right)$ to replace the sum of the output-oriented ratio $\sum(w \times \frac{y}{x})$ in computing the efficiency score in the DEA-R-I mathematical method, we suggest that the efficiency score of DEA-R-I will always be greater than or equal to the efficiency score of CCR-I (CCR input-oriented). This also means that the strategies of DEA-R-I are easier to achieve than the strategies of CCR-I because the strategy derived from the higher efficiency score needs fewer changes. If we can prove this hypothesis, the CCR-I model can be replaced by DEA-R-I because DEA-R-I provides a more accurate efficiency score in situations with weight restrictions and a better strategy in situations without weight restrictions.

Because of the above reasons, the first goal of this paper is to develop a mathematical method (we call it DEA-R-I) to derive the input-target improvement strategy in a situation with weight restrictions. The second goal is to prove that the input-target improvement strategy developed by DEA-R-I is always better than the CCR-I model in a situation without weight restrictions. Therefore, we can claim that the DEA-R-I model can replace the CCR-I model in all input-oriented situations.

2. Mathematical method to evaluate efficiency scores and derive input-target improvement strategies

Because there are no suitable input-oriented models for situations with weight restrictions on single I/O pairs, we developed a new model to evaluate the efficiency score and derive the input-oriented strategy. We applied a new model to calculate the efficiency score and then derive the input-target strategy from the efficiency score:

Step 1: Compute the efficiency score

DEA-R-I, the mathematical model for computing the efficiency score of a DMU object $\hat{\theta}_o$, is expressed as follows:

$$\text{Max } \hat{\theta}_o \quad (2)$$

$$\text{st.} \quad \sum_{i=1}^{m}\sum_{r=1}^{s} W_{ir}\frac{(X_{ij}/Y_{rj})}{(X_{io}/Y_{ro})} \geqslant \hat{\theta}_o \; j = 1,\ldots,n \quad (3)$$

$$\sum_{i=1}^{m}\sum_{r=1}^{s} W_{ir} = 1 \quad (4)$$

$$W_{ir} \geqslant 0, \quad \hat{\theta}_o \geqslant 0 \quad (5)$$

X_{ij}: ith input variable of the jth DMU.
Y_{rj}: rth output variable of the jth DMU.
X_{io}: ith input variable of the object.
Y_{ro}: rth output variable of the object.
W_{ir}: The weight of the ratio of $\frac{\text{the } i\text{th input variable } X_i}{\text{the } r\text{th output variable } Y_r}$.

$\sum_{i=1}^{m}\sum_{r=1}^{s} W_{ir}(X_{ij}/Y_{rj})/(X_{io}/Y_{ro})$: the relative efficient score with jth DMU's.

For each DMU object, the model first computes its relative efficiency score for each specified weight, and the smallest is selected as the efficiency score of this set of weights. Second, by adjusting the weighting set, a maximum efficiency score will be selected as the efficiency score $\hat{\theta}_o$ of the object. Since each DMU can get its optimal weight, we can say objectively that the DMU is inefficient if the efficiency score of this DMU is less than one. It is necessary to provide the improved strategy for this DMU, which we will discuss in next part. Using the data in Table 1 as an example, if we want to calculate the efficiency score of the DMU1 of Table 1, we must first find four relative efficiency scores for the DMU 1 in each weight.

When the weight set is $W_{11} = 1$ and $W_{12} = 0$, the relative efficiency scores of DMU 1 with DMUs 1–4 are $1 \times \left(\frac{2}{4}\right)/\left(\frac{2}{4}\right) + 0 \times \left(\frac{2}{3}\right)/\left(\frac{2}{3}\right) = 1.00, 1 \times \left(\frac{2}{3}\right)/\left(\frac{2}{4}\right) + 0 \times \left(\frac{2}{5}\right)/\left(\frac{2}{3}\right) = 1.33, 1 \times \left(\frac{2}{4.2}\right)/\left(\frac{2}{4}\right) + 0 \times \left(\frac{2}{4.2}\right)/\left(\frac{2}{3}\right) = 0.95$, and $1 \times \left(\frac{2}{5}\right)/\left(\frac{2}{4}\right) + 0 \times \left(\frac{2}{3}\right)/\left(\frac{2}{3}\right) = 0.80$, respectively (the right-most points of lines 1, 2, 3, and 4 shown in Fig. 1a). The relative efficiency score of DMU 1 with DMU 4 in weight $W_{11} = 1$ and $W_{12} = 0$ is 0.8, which means that if we need one unit of X_1 from DMU 1 to produce one unit of Y_1, only 0.8 units of X_1 from DMU 4 are needed to produce one unit of Y_1. Repeating the computation, the relative efficiency scores of DMU1 with each DMU in different weight sets are shown in Fig. 1a. In Fig. 1a, when W_{11} is between 0.000 and 0.231, the lowest value of the four relative efficiency scores is the relative efficiency score with DMU 2.

Table 1
One input and two-outputs.

DMU	Input	Output	
	X_1	Y_1	Y_2
1(A)	2.0	4.0	3.0
2(B)	2.0	3.0	5.0
3(C)	2.0	4.2	4.2
4(D)	2.0	5.0	3.0

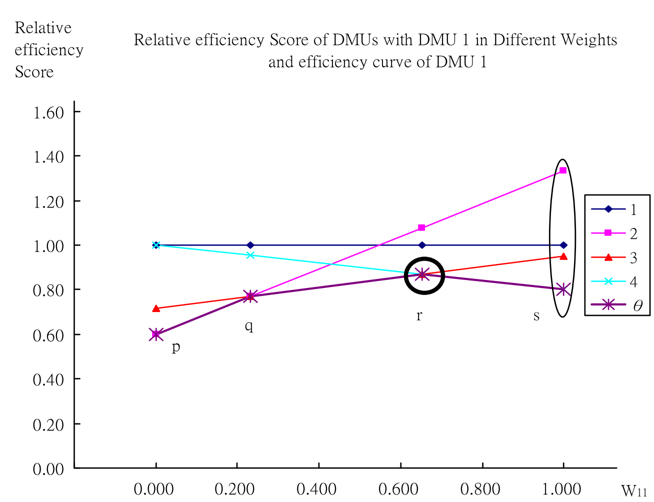

Fig. 1a. Relative Efficiency Score of DMU 1 with DMUs in Different Weights and the Efficiency curve of DMU 1.